Black Hole Astrophysics
Chapters Ch13.1~13.1.2.2

All figures extracted from online sources of from the textbook.

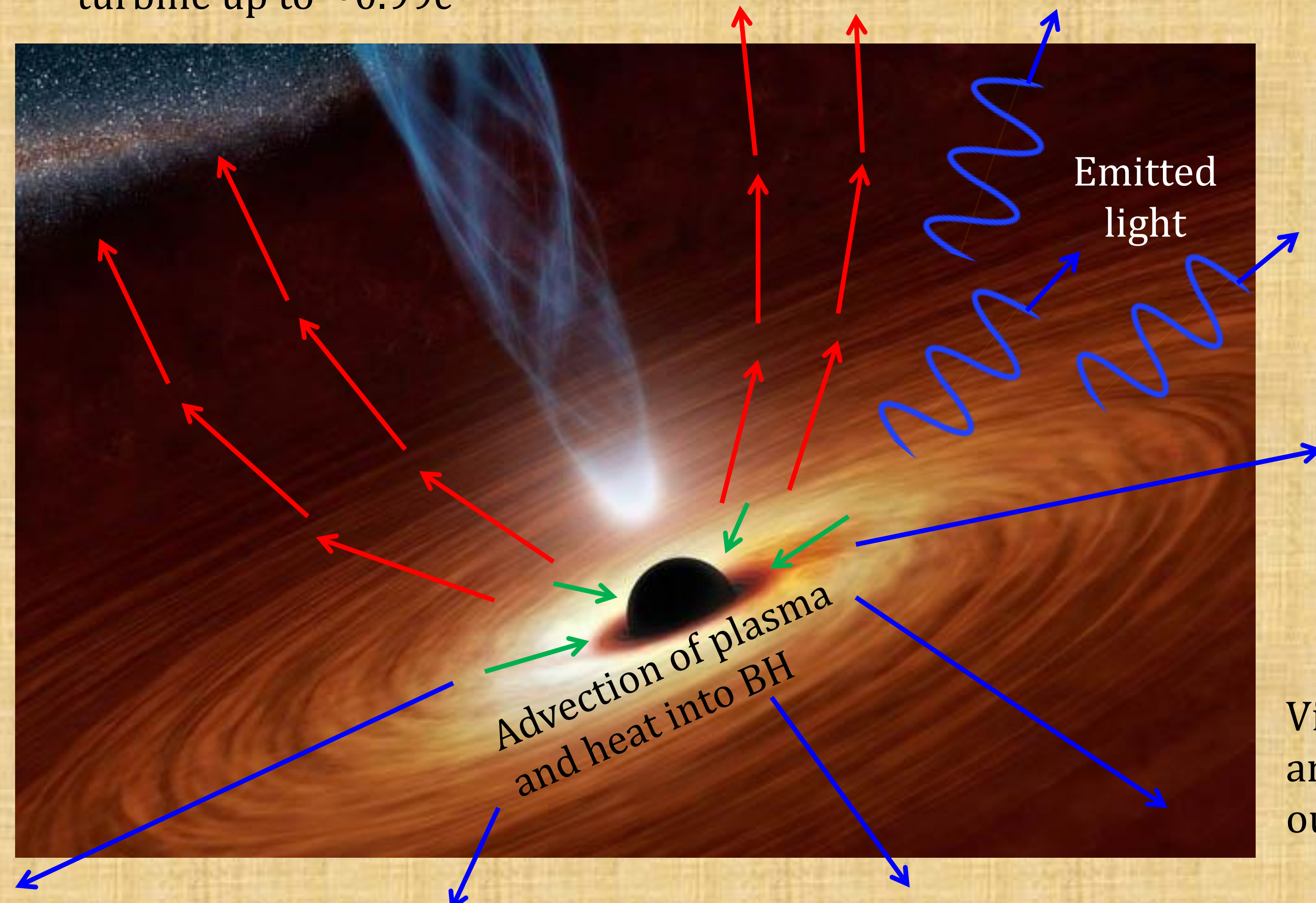
A word of notice

As I have skipped chapters 9.5~12, when the title of the slide contains "was already discussed *(in the book)*" most likely it is something that I skipped!

Also, this is the first time I studied any of these materials so please correct me if you find anything that I say defying logic.

The five exhaust systems

As was explained a few weeks ago, there are 5 exhaust systems that remove matter/energy from the BH combustion chamber.

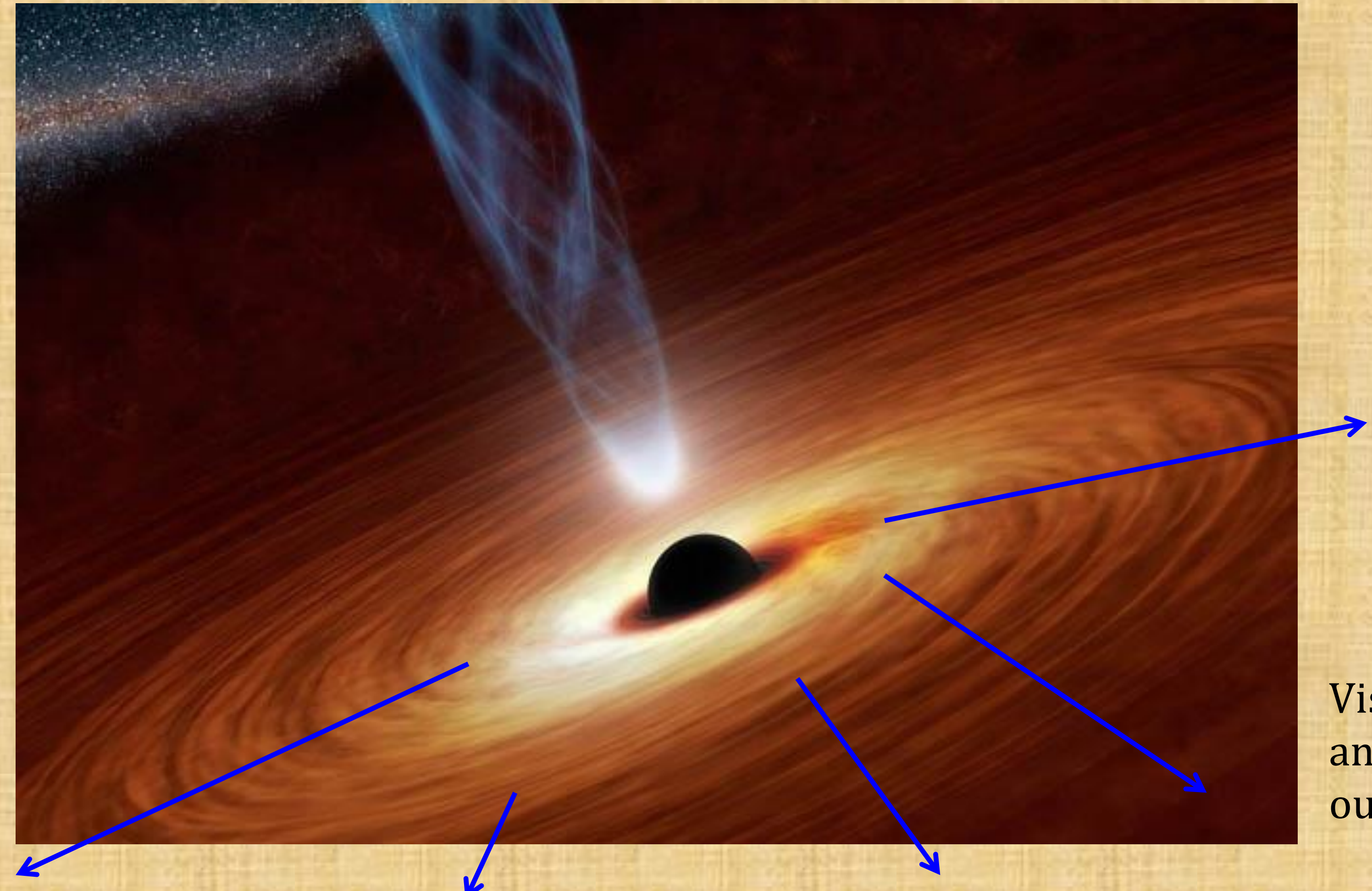


What was already discussed (in the book)

Photons produced by viscous heating *removes heat and binding energy* from the fuel and keeps it at low temperature so it doesn't explode out of the accretion disk combustion chamber.

What was already discussed (in the book)

Viscous transport of angular momentum via the Magneto-Rotational Instability (MRI) removes angular momentum from the material close to the BH thus allows them to sink further in and generate more energy.



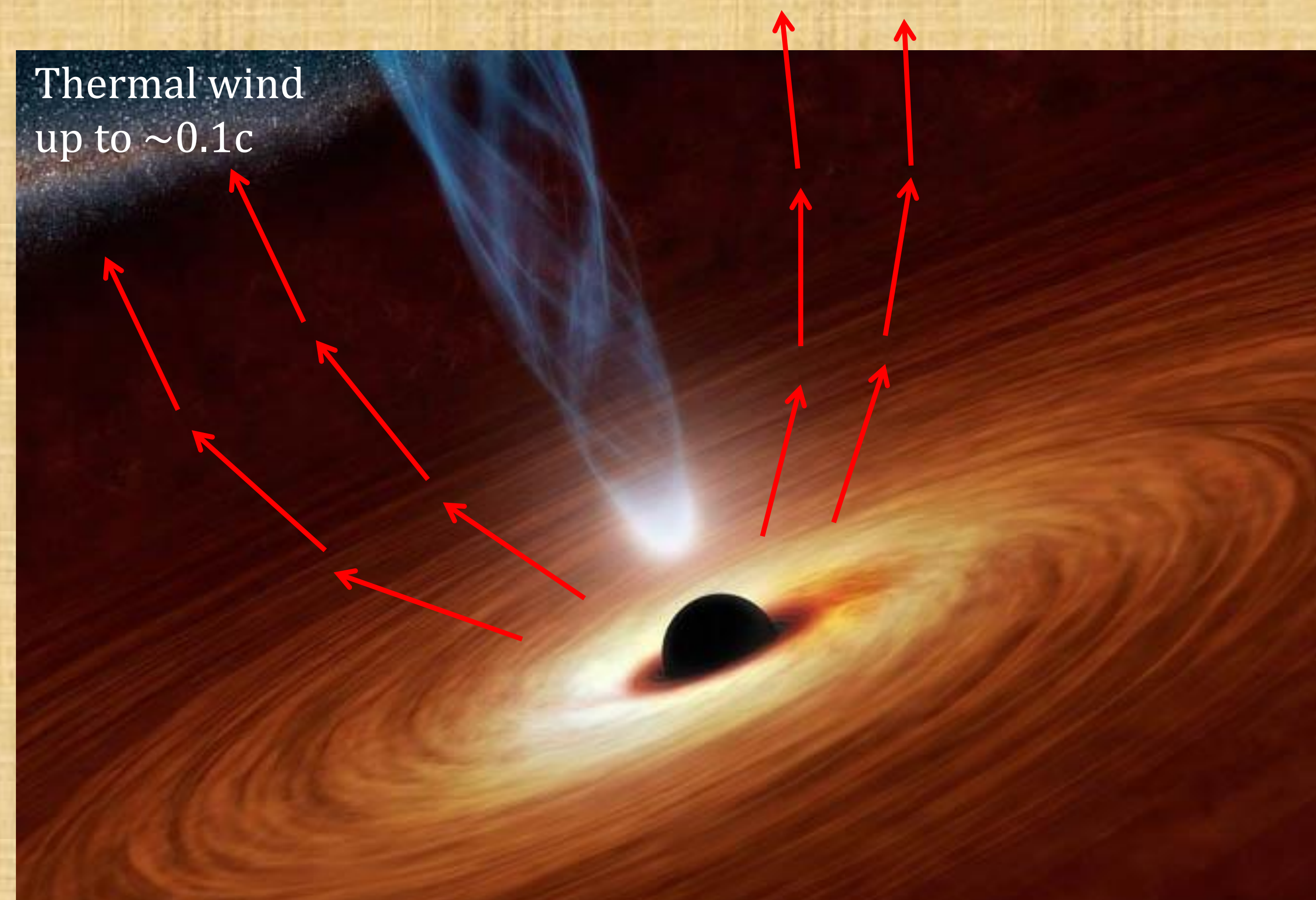
Viscous Transport of angular momentum outward in disk

What was already discussed (in the book)

Material (and heat) that has sunk into the hole becomes part of the BH and no longer has an effect the workings of the BH engine. The only thing it does now is to help the BH grow.

What is going to be discussed

When the rate of accretion becomes so large that the pressure of radiation becomes important, then radiation-pressure-driven winds help to self-limit the amount of material that is allowed to fall toward the black hole or neutron star.



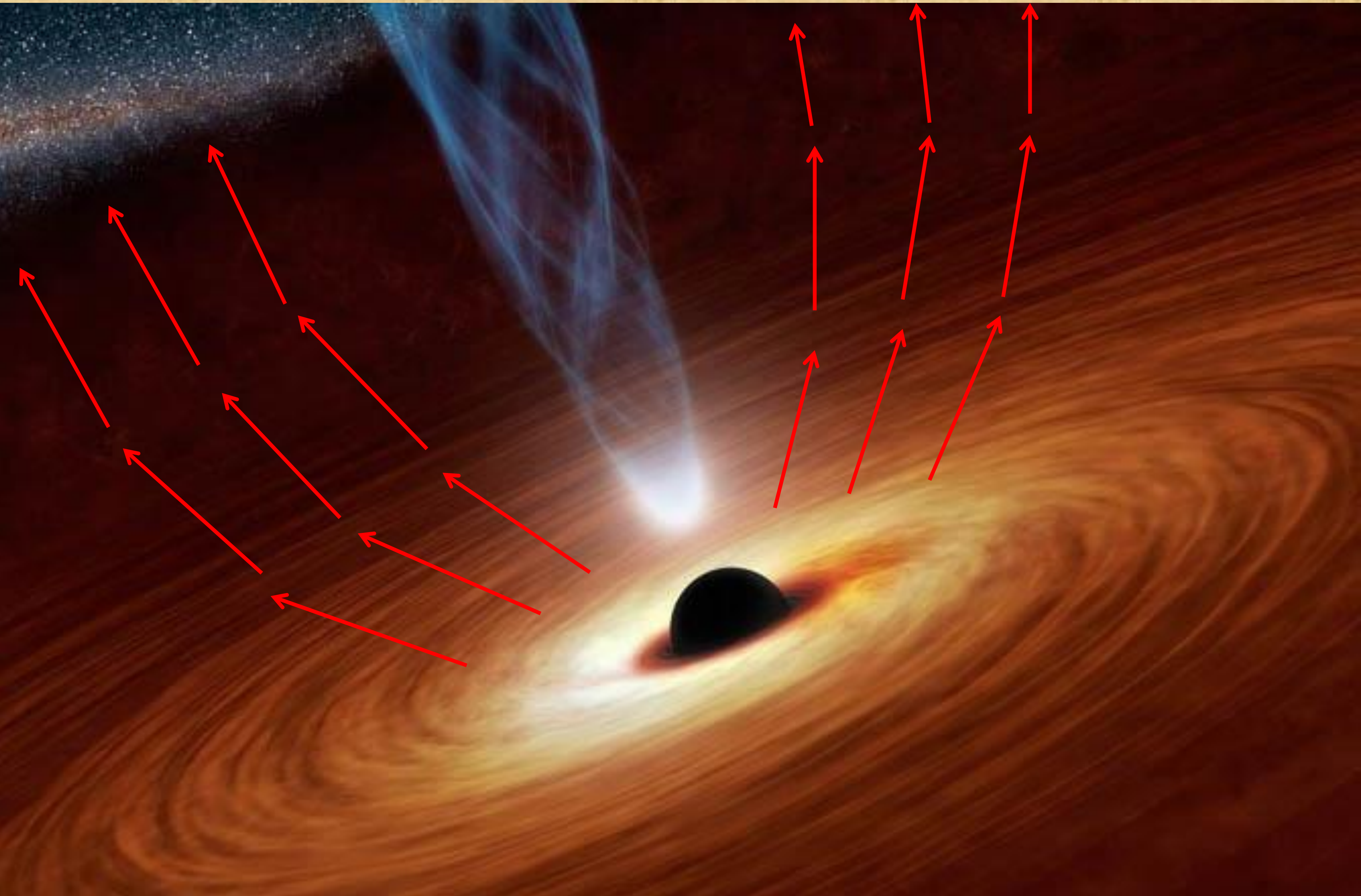
It can also be done in a steady manner that allows the engine to fuel steadily but at a reduced rate.

What would be discussed

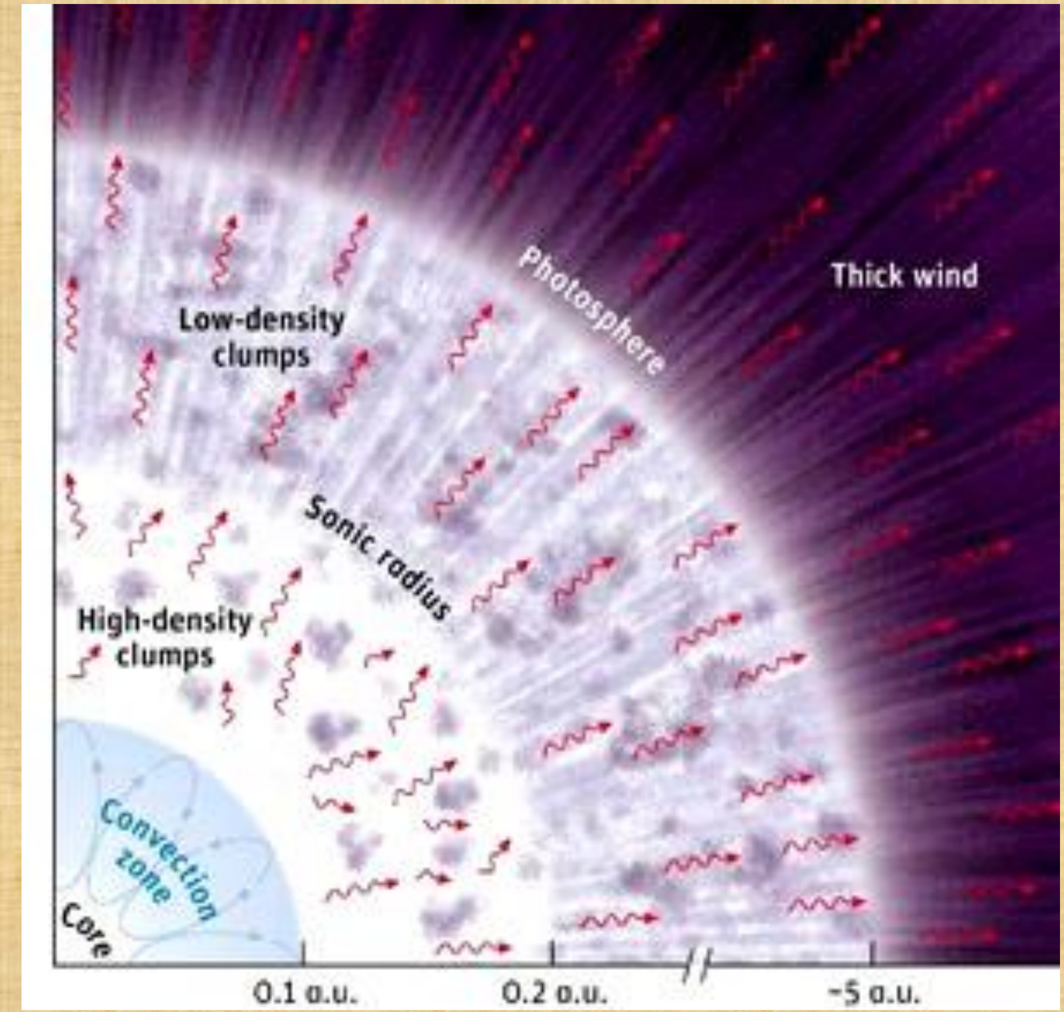
Winds and jets of nonthermal particles driven by a magnetic turbine up to ~0.99c

The last exhaust system also removes angular momentum and involves the complicated action of magnetic fields. It will be discussed in depth in Ch14~Ch15.

Today：Radiation driven winds



Radiation driven winds - Intro



For radiation to be capable of removing stuff from a system, the easiest way would simply to have the radiation force exceed gravity.

i.e.

$$a_r^{e-} = \frac{\kappa_{es} L_{rad}}{4\pi c r^2} \equiv \ell_{rad} \frac{GM}{r^2}$$

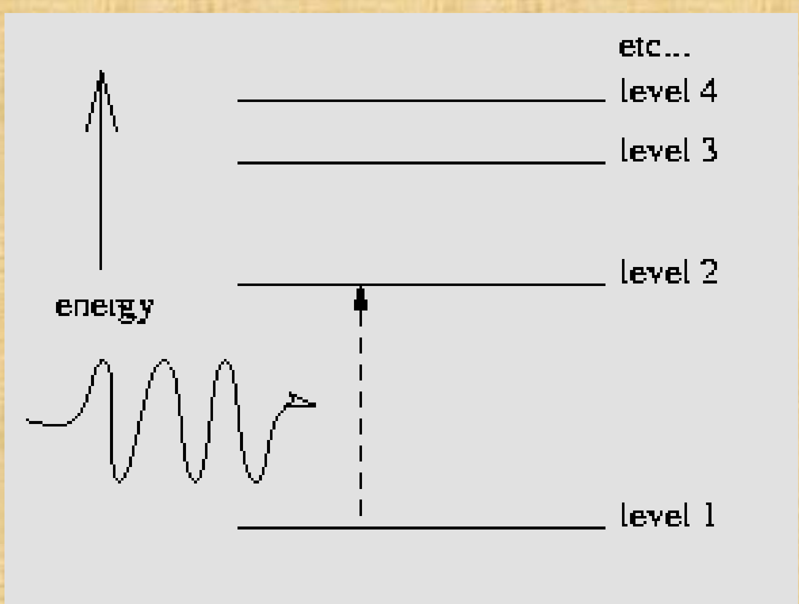
$$\ell_{rad} \geq 1$$

L_{rad} is the luminosity
κ_{es} electron scattering opacity
ℓ_{rad} the Eddington ratio

Radiation driven winds - Intro

However, it actually isn't necessary to exceed the Eddington limit for strong winds to develop. With the assistance of bound-bound (line) absorption, we can have matter being accelerated outward by radiation that is sub-Eddington

$$a_r^{\text{lines}} = \frac{\kappa_{\text{lines}} L_{\text{rad}}}{4\pi c r^2} \equiv (1 + \mathcal{M})\ell_{\text{rad}} \frac{\text{GM}}{r^2} \geq \frac{\text{GM}}{r^2}$$



$1 + \mathcal{M}$, the force multiplier is due to additional acceleration from line absorption. Or simply

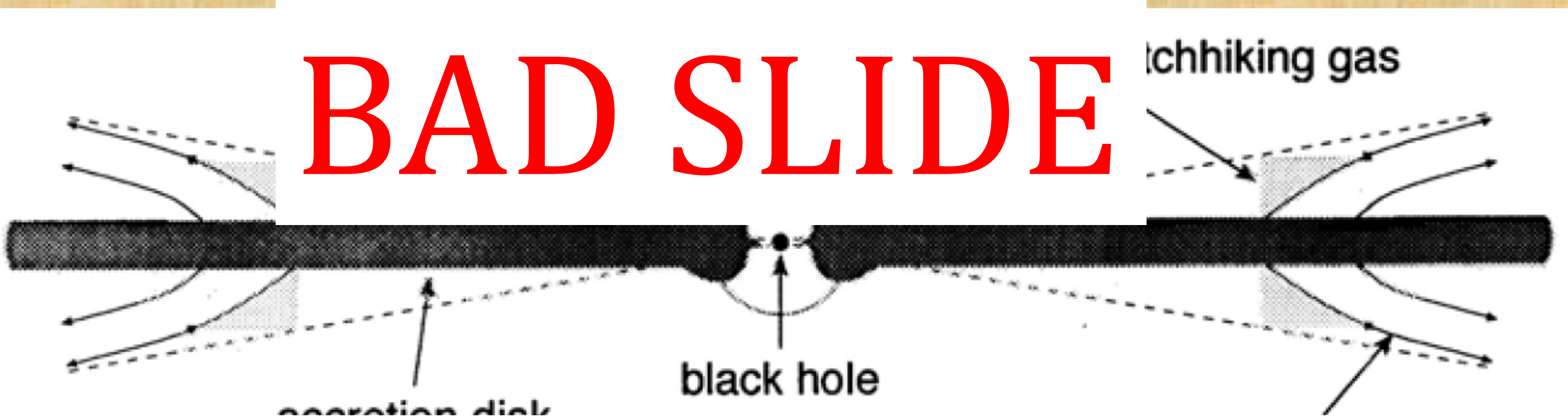
$$1 + \mathcal{M} = \frac{\text{momentum transfer by line absorption}}{\text{momentum transfer by scattering}}$$

The same process occurs in massive star atmospheres, and line opacity-driven winds are believed to be the cause of strong outflows emanating from O and B stars. They likely will play a role in black hole engines as well.

Model by Murray and Chiang

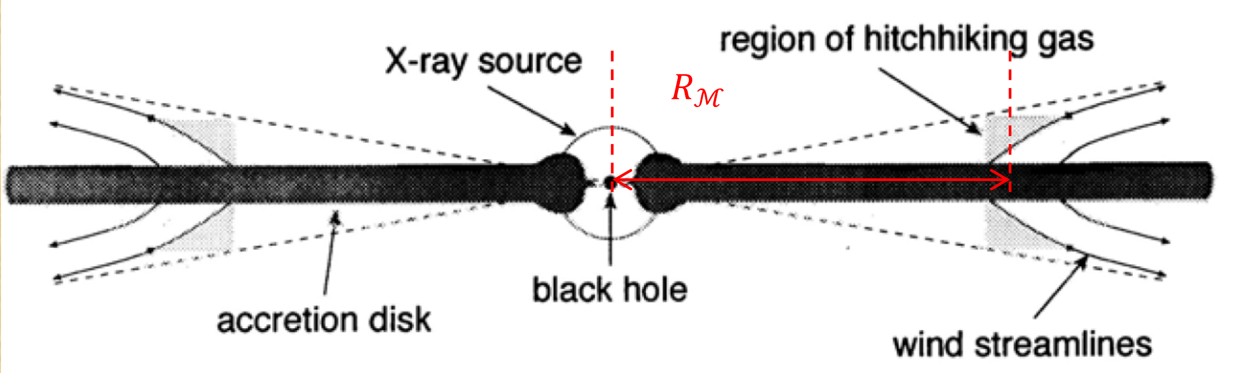
A bit of history:
Several models have been proposed for line-driven winds from black hole accretion disks. One model that <u>*potentially can explain both the broad emission and absorption lines*</u> seen in quasars and Seyfert nuclei was developed in 1995 by Norman Murray, James Chiang, and their colleagues.

BAD SLIDE



the wind was shown earlier in Fig. 2.16(bottom) in Chapter 2. When the accretion rate in the disk is close to the Eddington rate ($\dot{m} > 0.2$), the "inner" disk is rather thick ($H \sim constant \propto \dot{m}$; see Section 12.2.2). In addition, if the "inner" region is thermally unstable, it will spend some of its Lightman–Eardley limit cycle time in a very thick ($H \sim R$) ADAF-like state as a hard X-ray source ($T \sim 10^8$ K). Beyond the "inner" region, the disk is flared ($H \propto R$), allowing its atmosphere to see the luminous disk center.

The wind launch radius



At a radius $R_{\mathcal{M}}$ where the surface temperature of the disk is *cool enough to have partially-ionized atoms* $T_* \simeq 10^{4-5}K$, the atomic line absorption opacity will be large enough to give a large force multiplier $\mathcal{M}$.

An equation (some model for disk temperature) from previous chapters gives this radius:

$$R_{\mathcal{M}} = (5000\sim10^5) r_g m^{-0.25} \dot{m}^{0.25} \approx (7.5 \times 10^8 \sim 1.5 \times 10^{10}) m^{0.75} \dot{m}^{0.25} \text{cm}$$

Object	Radius $R_{\mathcal{M}}$ (cm)
$1.4\ M_\odot$ Neutron Star	5×10^9
$10^9 M_\odot$ Black Hole	$2 \times 10^{16} \sim 100 r_g$

Adiabatic Wind

Mass Conservation

$$\frac{d}{dr}(4\pi r^2 \rho v) = 0 \rightarrow \frac{2}{r} + \frac{1}{\rho}\frac{d\rho}{dr} + \frac{1}{v}\frac{dv}{dr} = 0 \rightarrow \frac{1}{\rho}\frac{d\rho}{dr} = -\left(\frac{2}{r} + \frac{1}{v}\frac{dv}{dr}\right)$$

Momentum Equation

$$v\frac{dv}{dr} = -\frac{1}{\rho}\frac{dp}{dr} - \frac{GM}{r^2}$$

Adiabatic E.O.S.

$$p = K\rho^\gamma \rightarrow \frac{dp}{dr} = K\gamma\rho^{\gamma-1}\frac{d\rho}{dr} = \frac{\gamma p}{\rho}\frac{d\rho}{dr} = c_s^2\frac{d\rho}{dr}$$

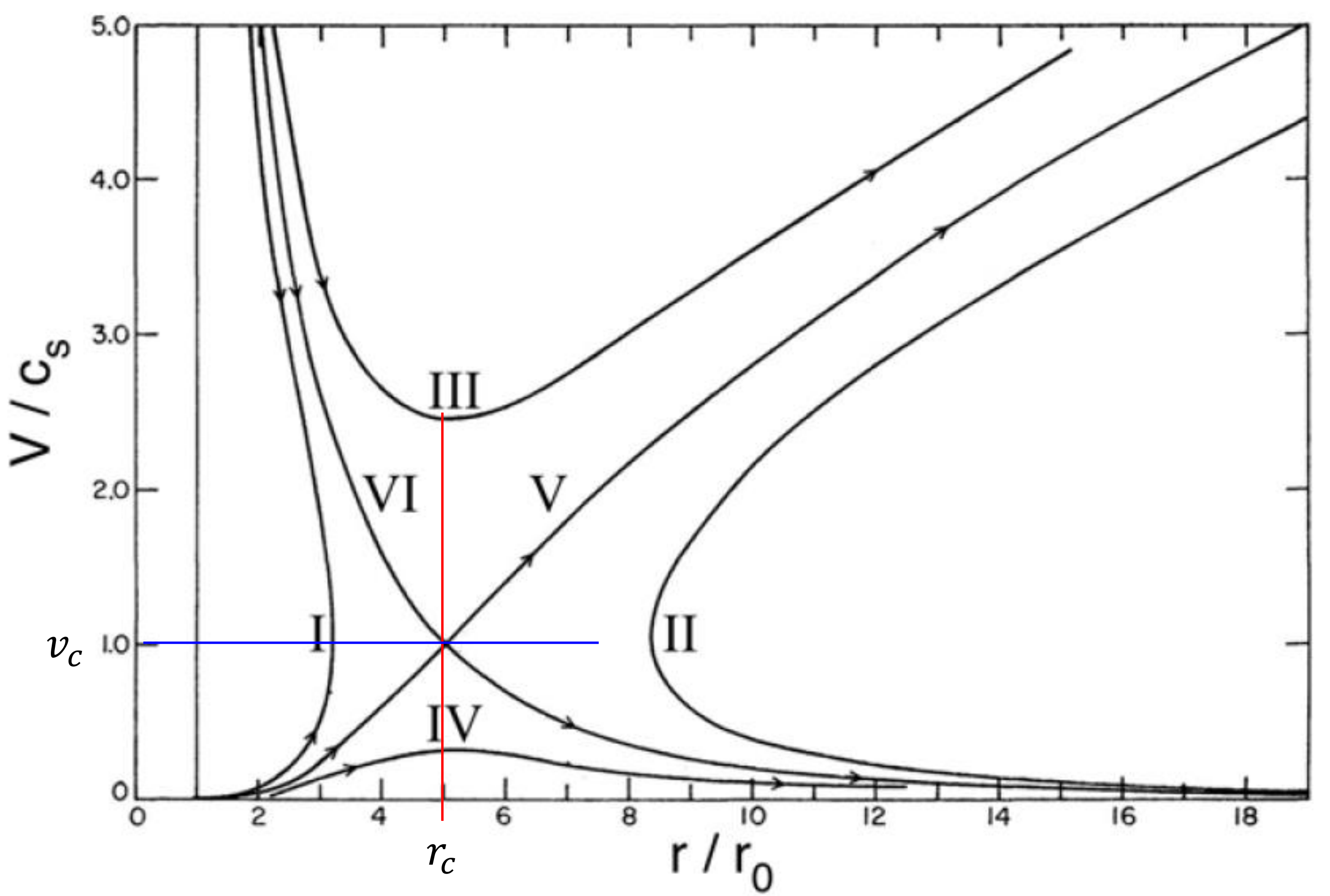
$$v\frac{dv}{dr} = -\frac{1}{\rho}\frac{dp}{dr} - \frac{GM}{r^2} = -c_s^2\left(\frac{1}{\rho}\frac{d\rho}{dr}\right) - \frac{GM}{r^2} = c_s^2\left(\frac{2}{r} + \frac{1}{v}\frac{dv}{dr}\right) - \frac{GM}{r^2}$$

$$(v^2 - c_s^2)\frac{r}{v}\frac{dv}{dr} = \left(2c_s^2 - \frac{GM}{r}\right)$$

$$\frac{d\ell n(v)}{d\ell n(r)} = -\left(\frac{GM/r - 2c_s^2}{v^2 - c_s^2}\right)$$

Adiabatic Wind

$$(v^2 - c_s^2)\frac{r}{v}\frac{\mathrm{dv}}{\mathrm{dr}} = \left(2c_s^2 - \frac{\mathrm{GM}}{r}\right) \qquad v_c = \pm\, c_s \text{ at } r_c = \frac{\mathrm{GM}}{2c_s^2}$$



Adiabatic Wind

Mass Conservation

$$\frac{d}{\mathrm{dr}}(4\pi r^2 \rho v) = 0 \rightarrow \frac{2}{r} + \frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{dr}} + \frac{1}{v}\frac{\mathrm{dv}}{\mathrm{dr}} = 0 \rightarrow \frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{dr}} = -\left(\frac{2}{r} + \frac{1}{v}\frac{\mathrm{dv}}{\mathrm{dr}}\right)$$

Momentum Equation

$$v\frac{\mathrm{dv}}{\mathrm{dr}} = -\frac{1}{\rho}\frac{\mathrm{dp}}{\mathrm{dr}} - \frac{GM}{r^2}$$

Adiabatic E.O.S.

$$p = K\rho^{\gamma} \rightarrow \frac{\mathrm{dp}}{\mathrm{d}r} = K\gamma\rho^{\gamma-1}\frac{\mathrm{d}\rho}{\mathrm{d}r} = \frac{\gamma p}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}r} = c_s^2\frac{\mathrm{d}\rho}{\mathrm{d}r}$$

$$v\frac{\mathrm{dv}}{\mathrm{dr}} = -\frac{1}{\rho}\frac{\mathrm{dp}}{\mathrm{dr}} - \frac{GM}{r^2} = -c_s^2\left(\frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{d}r}\right) - \frac{GM}{r^2} = c_s^2\left(\frac{2}{r} + \frac{1}{v}\frac{\mathrm{dv}}{\mathrm{dr}}\right) - \frac{GM}{r^2}$$

$$(v^2 - c_s^2)\frac{r}{v}\frac{\mathrm{dv}}{\mathrm{dr}} = \left(2c_s^2 - \frac{GM}{r}\right)$$

$$\frac{d\ell\mathrm{n}(v)}{d\ell\mathrm{n}(r)} = -\left(\frac{GM/r - 2c_s^2}{v^2 - c_s^2}\right)$$

Radiation (line) driven wind

Mass Conservation

$$\frac{d}{\mathrm{dr}}(4\pi r^2 \rho v) = 0 \rightarrow \frac{2}{r} + \frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{dr}} + \frac{1}{v}\frac{\mathrm{dv}}{\mathrm{dr}} = 0 \rightarrow \frac{1}{\rho}\frac{\mathrm{d}\rho}{\mathrm{dr}} = -\left(\frac{2}{r} + \frac{1}{v}\frac{\mathrm{dv}}{\mathrm{dr}}\right)$$

Momentum Equation

$$v\frac{\mathrm{dv}}{\mathrm{dr}} = -\frac{1}{\rho}\frac{\mathrm{dp}_g}{\mathrm{dr}} - \frac{\mathrm{GM}}{r^2} + \underline{(1+\mathcal{M})\ell_{\mathrm{rad}}\frac{\mathrm{GM}}{r^2}} = -\frac{1}{\rho}\frac{\mathrm{dp}_g}{\mathrm{dr}} - (1-(1+\mathcal{M})\ell_{\mathrm{rad}})\frac{\mathrm{GM}}{r^2}$$

Energy Equation

Implicitly, above assumes $\frac{1}{r^2}\frac{d(r^2 H_r)}{\mathrm{dr}} = 0$

Generally, assuming $p_g = a^2\rho$, $\frac{1}{\rho}\frac{\mathrm{dp}_g}{\mathrm{dr}} = \frac{1}{\rho}\frac{\mathrm{dp}_g}{\mathrm{dr}} = \frac{a^2}{\rho}\frac{\mathrm{d}\rho}{\mathrm{dr}} + \underline{\frac{\mathrm{da}^2}{\mathrm{dr}}}$ Extra term

$$v\frac{\mathrm{dv}}{\mathrm{dr}} = a^2\left(\frac{2}{r} + \frac{1}{v}\frac{\mathrm{dv}}{\mathrm{dr}}\right) - \frac{\mathrm{da}^2}{\mathrm{dr}} - (1-(1+\mathcal{M})\ell_{\mathrm{rad}})\frac{\mathrm{GM}}{r^2}$$

$$(v^2 - a^2)\frac{r}{v}\frac{\mathrm{dv}}{\mathrm{dr}} = \left(2a^2 - \frac{1}{r}\frac{\mathrm{da}^2}{\mathrm{dr}} - (1-(1+\mathcal{M})\ell_{\mathrm{rad}})\frac{\mathrm{GM}}{r}\right)$$

$$\frac{d\ell \mathrm{n}(v)}{d\ell \mathrm{n}(r)} = -\left(\frac{(1-(1+\mathcal{M})\ell_{\mathrm{rad}})\,\mathrm{GM}/r - 2a^2 + \mathrm{da}^2/\mathrm{d}\ell\mathrm{n}\,(r)}{v^2 - a^2}\right)$$

Line driven disk wind equation

Line driven disk wind equation

$$\frac{d\ell n(V)}{d\ell n(r)} = -\left\{\frac{1-\left[(1+\mathcal{M})\tilde{\ell}_{\text{rad}}\right]\left(\frac{f\,GM}{r}\right) - 2a^2 + \frac{da^2}{d\ell n(r)}}{v^2 - a^2}\right\}$$

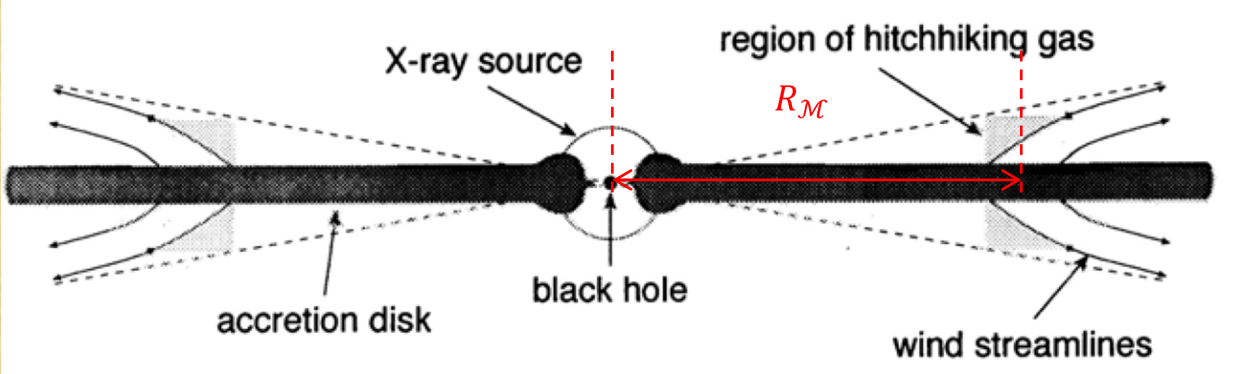
Basic wind equation

$$\frac{d\ell n(V)}{d\ell n(r)} = -\left(\frac{\left(\frac{GM}{r}\right) - 2{c_s}^2}{v^2 - {c_s}^2}\right)$$

$a = \sqrt{\frac{p_g}{\rho}}$ the isothermal gas pressure. It is the use of 'a' instead of c_s that adds the gradient term $\frac{da^2}{d\ell n(r)}$

The Eddington ratio ℓ_{rad} is now replaced by $\tilde{\ell}_{\text{rad}} \equiv \ell_{\text{rad}} e^{-\tau_{\text{es}}}$ to account for disk geometry.

$f \equiv 1 - R_{\mathcal{M}}/r$ causes the wind to start at zero velocity at radius $R_{\mathcal{M}}$.



The singular point for line driven winds

$$\frac{d\ell n(V)}{d\ell n(r)} = -\left\{\frac{1 - \left[(1+\boxed{\mathcal{M}})\tilde{\ell}_{\text{rad}}\right]\left(\frac{\text{f GM}}{r}\right) - 2a^2 + \boxed{\frac{da^2}{d\ell n(r)}}}{v^2 - a^2}\right\}$$

The singular point is not at v=a because there are two extra derivatives on the right (if brought over to the left would modify the denominator $v^2 - a^2$ to something else.)

The force multiplier also contains as derivative!

Recall that $1 + \mathcal{M} = \frac{\text{momentum transfer by line absorption}}{\text{momentum transfer by scattering}}$

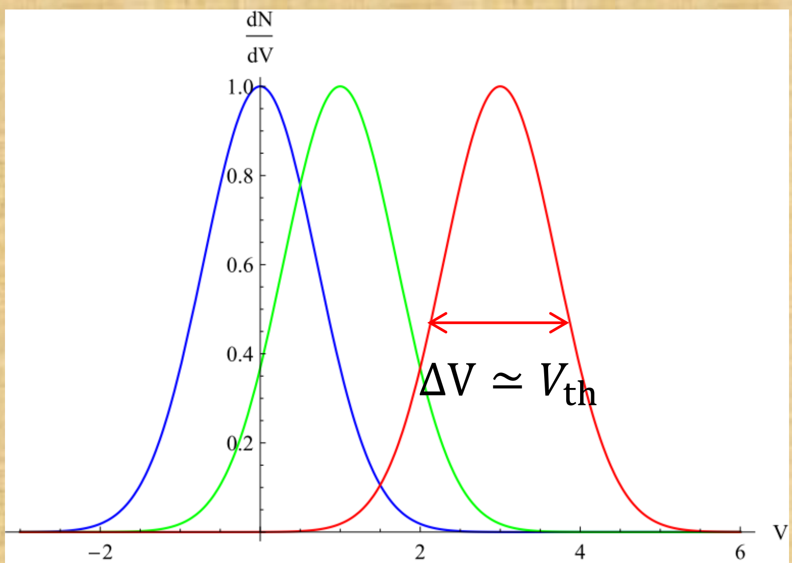
Thus, $\mathcal{M}$ is only large when line absorption is active!

We will explain this on the next few slides.

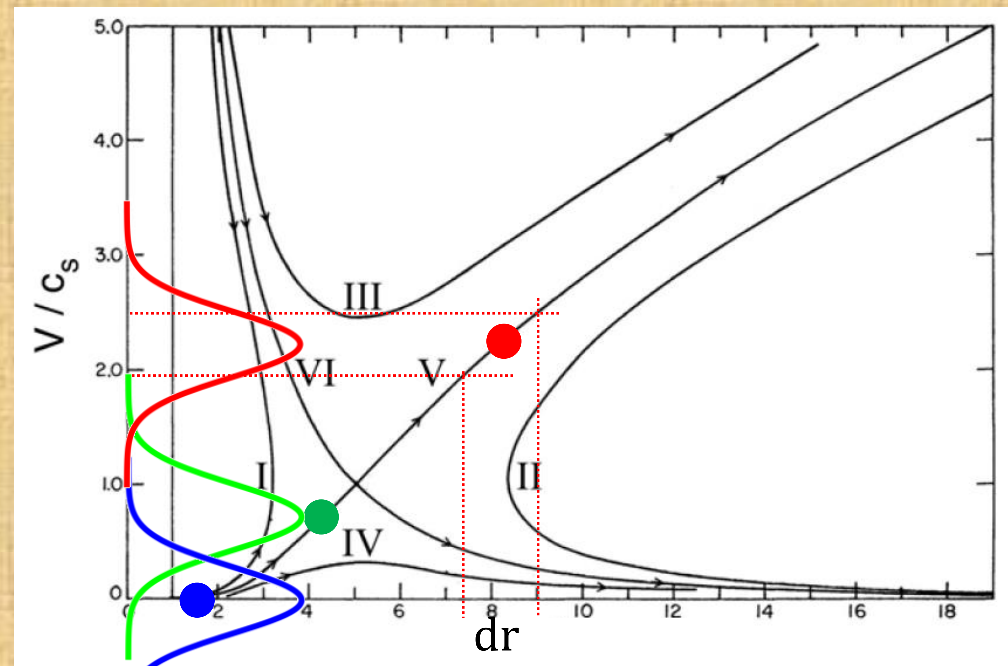
Position dependence of $\mathcal{M}$

Since the wind is accelerating, radiation will be Doppler shifted with respect to the atoms in the wind

At different radii, we will have a different central velocity.



Let's consider 3 points along the curve



Therefore, we must be in a region (some dr) in which τ_L is large for the action to occur.
This tells us that $\mathcal{M}$ also contains a derivative

$$\mathcal{M}(\tau_L) = \mathcal{M}\left(\kappa_L \rho \frac{V_{th}}{|dV/dr|}\right)$$

The terminal velocity

The actual terminal velocity obviously depends on the exact functional form of $\mathcal{M}$, but a rough approximation can be found to be:

$$v_\infty \approx \sqrt{\frac{4GM}{R_\mathcal{M}}}$$

And the model seems to work:

Object	$v_\infty(\mathbf{km}\cdot s^{-1})$	Source	Observed Speed ($\mathbf{km}\cdot s^{-1}$)
1.4 $M_\odot$ Neutron Star	~3000	NS X-ray binary Circinus X-1	2000
$10^9 M_\odot$ Black Hole	~25,000	Broad Absoprtion Line (BAL) QSOs	5000~50,000

The model also predicts broad emission lines (BELs) interior to the BALs with Keplerian widths similar to those at $R_\mathcal{M}$. That is, the BEL gas is predicted to be the atmospheric gas of the disks, gas before it is significantly accelerated into a wind. These work out to about $V_\infty/2$ or $\sim 1500\,\mathrm{km\,s^{-1}}$ in Cir X-1 and $12,000\,\mathrm{km\,s^{-1}}$ in the BAL QSO case. These also are typical of the emission lines in these objects.

The X-Ray Heating problem

Accreting NS/BHs are strong X-ray sources with radiation temperature $T_r \sim 10^8 K$. Why does a model with accelerated gas being only of $10^{4-5} K$ work well?

This X-ray radiation is believed to originate near the center of the accretion disk from a corona above the cooler accretion disk. These hard photons should raise the temperature of the disk surface by Compton scattering the cooler electrons to an equilibrium "Compton" temperature of

$$T_C \sim \frac{1}{4} T_r \approx 2.5 \times 10^7$$

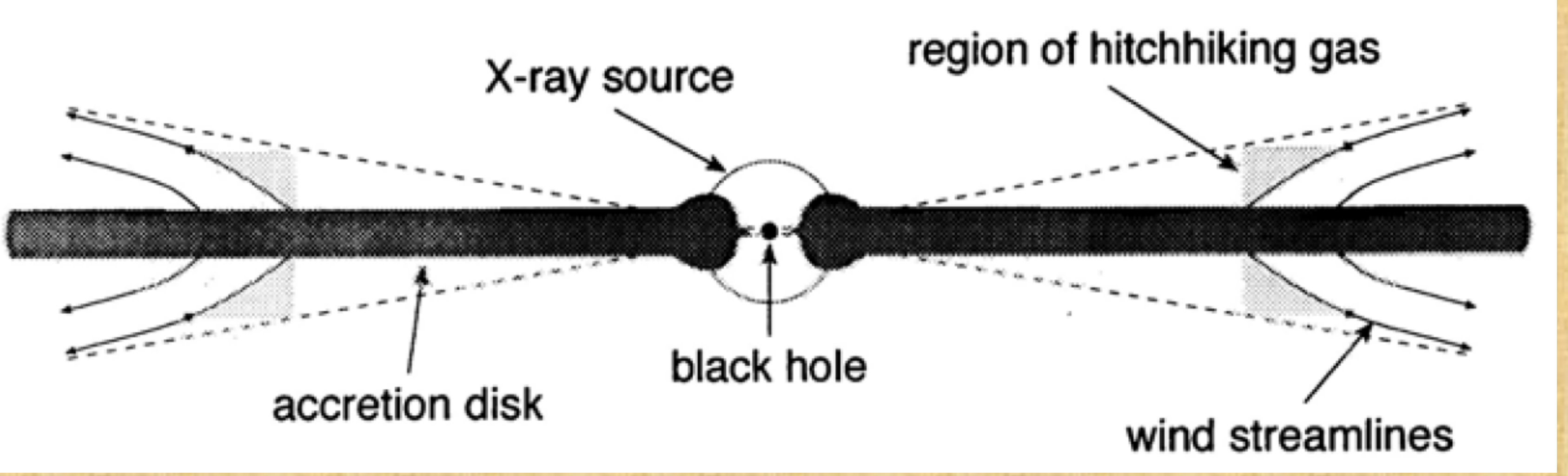
However, the observed winds are nowhere nearly that hot.

Source	Line temperature (K)
NS X-ray binary Circinus X-1	$\sim 10^6$
Broad Absoprtion Line (BAL) QSOs	$\sim 10^{4-5}$

X-ray Heating problem–Explanation 1

Main idea: Shielding gas

Murray and Chiang suggested that cool, dense gas is drawn up from the surface of the disk, just inside the launching point, by the wind itself. This is thought to be accomplished by pressure gradients induced by the wind loss. This "hitchhiking gas" then shields the accelerating outflow from the X-rays but not from the UV photons that are doing the acceleration.



X-ray Heating problem–Explanation 2

Main idea: Shielding gas

Another possibility is that Compton heating of the disk surface interior to $R_{\mathcal{M}}$ produces the shielding gas – in the form of a cool corona. That corona would attenuate X-rays from the central source, allowing a wind to be launched from a radius R_C (beyond which Compton heating of the surface would be unimportant). R_C would differ from $R_{\mathcal{M}}$, of course, and it is not known at this time whether this would give as successful a model as the standard Murray and Chiang model.

X-ray Heating problem–Explanation 3

Main idea: State transition

The presence of strong Compton-heating X-ray emission comes from the assumption that sources that show winds are simply normal objects, viewed along the surface of the disk. A third possibility, however, is that Cir X-1 and BAL QSOs are not like other X-ray sources and quasars.

Indeed, they are thought to be radiating very close to the Eddington limit ($\ell_{\mathrm{rad}} > 0.5$), whereas quasars are thought to be somewhat less luminous ($\ell_{\mathrm{rad}} \sim 0.1$)

When $\dot{m} \rightarrow 1$, the accretion flow may enter a cooler, optically thick, advectively-cooled "slim" disk state. That is, the inner hot corona that normally produces the X-rays may be quenched by the geometrically thick inflow. This would leave only the cooler disk accretion itself, implying central temperatures from equation (12.52) of $\sim 10^7$K for Cir X-1 and $\sim 10^5$K for BAL QSOs.

X-ray Heating problem–Explanation 4

Main idea: The outflow is not line-driven!

In Section 16.4.1 we suggest yet another explanation: the outflow is not line-driven at all but, rather, an advection-dominated inflow–outflow solution (ADIOS) wind, driven by thermal pressure of the hot component in the two-phase medium that may develop in the unstable Shakura–Sunyaev “inner” disk region (Section 12.2.2). (ADIOS winds will be discussed later in this chapter in Section 13.2.)

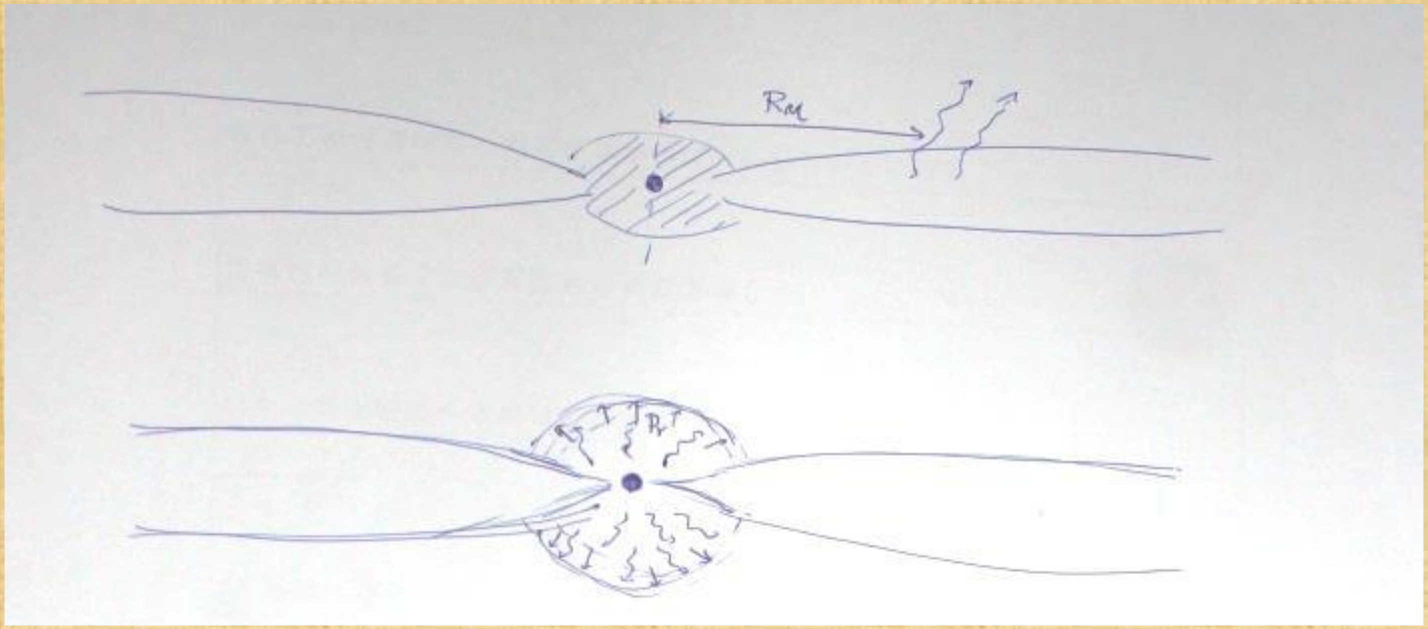
Again, it is the cool component that hitchhikes with the hot component, and it is lifted off the surface of the inflow by pressure gradients, but the latter develop simply because the ADIOS outflow is naturally unbounded to begin with.

Continuum-Driven, Super-Eddington Winds (Ch13.1.2)

Continuum-Driven, Super-Eddington Winds
Introduction

When the disk accretion rate exceeds the Eddington limit ($\dot{m} > 1$), then the wind takes on quite a different character.

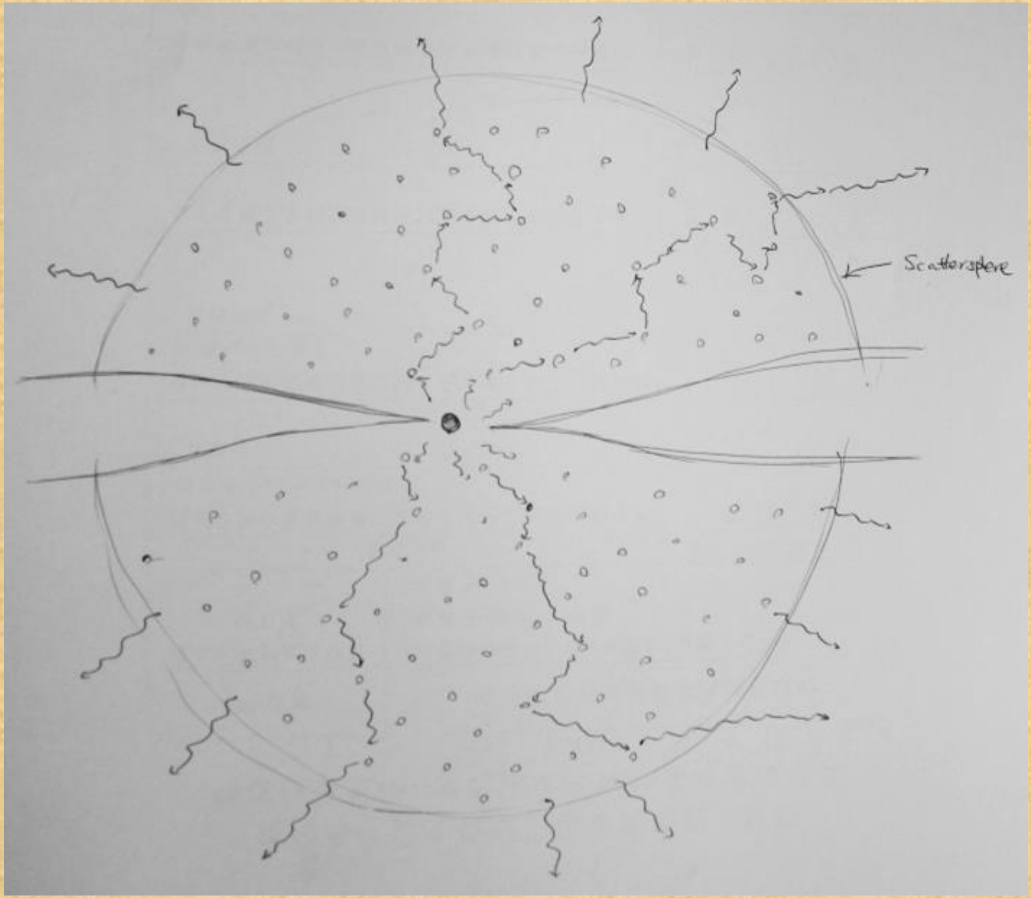
Instead of being driven from the surface of the disk "middle" or "outer" region, the radiation force on the electrons in the "inner" disk alone potentially can exceed gravity ($\ell_{\rm rad} > 1$). The "inner" region of the disk puffs up into a thick, radiation-pressure-supported, toroidal (almost spherical) structure.



Continuum-Driven, Super-Eddington Winds
Introduction

If the luminosity generated exceeds L_{Edd}, an optically thick, essentially spherical wind will be driven from the disk "inner" region.

Generally the wind will be so dense that, even for moderately high super-Eddington accretion rates ($\dot{m} = 10 \sim 100$), the surface of last scattering or "scattersphere" will be at a radius of $10^{3-5} r_g$. This wind, therefore, could sometimes engulf a large fraction of the entire accretion disk, making the accreting black hole look more like a luminous nova than an X-ray source.

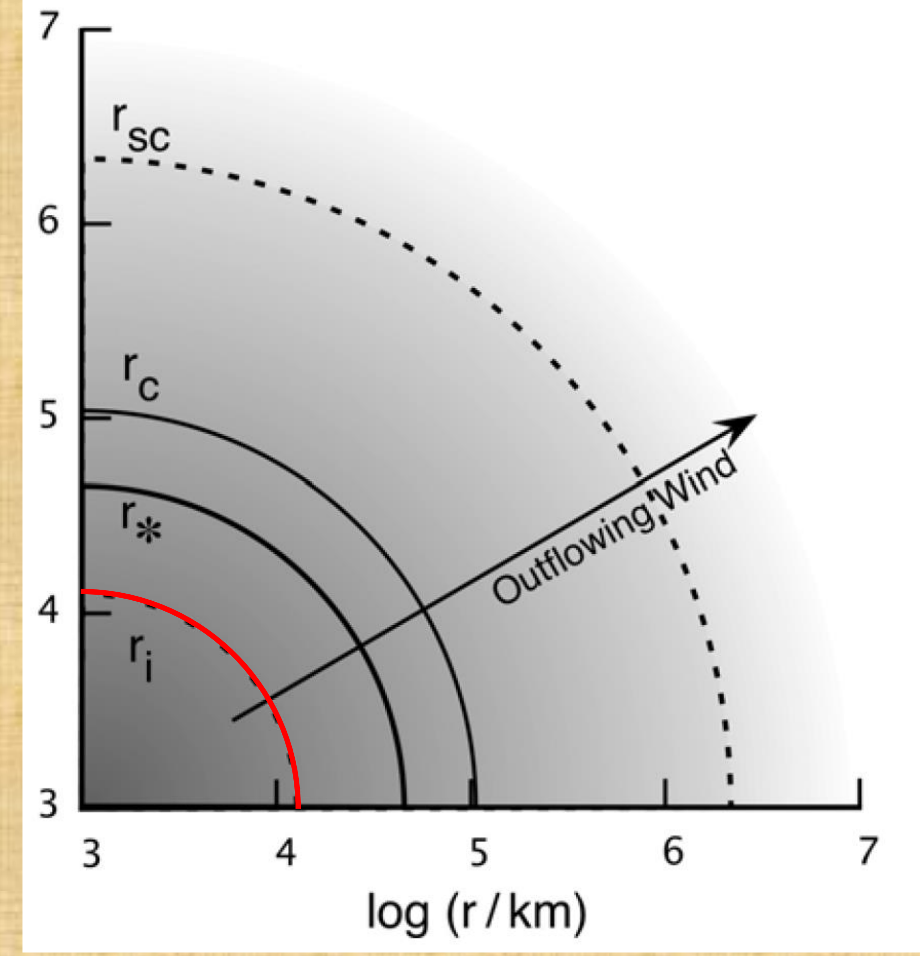


*Note that the following model mainly follows that developed in the paper by the same author in 1982.

Physical Structure – Intro

The Injection Radius (r_i):

Near this radius, $r_i = 8.2 \times 10^5\ m \cdot \dot{m}$ cm the disk is bloated by radiation pressure.



In our model we will assume that all the material exiting the system in the wind is injected at r_i , although in reality some of it comes from the region r < r_i and perhaps a little beyond as well.

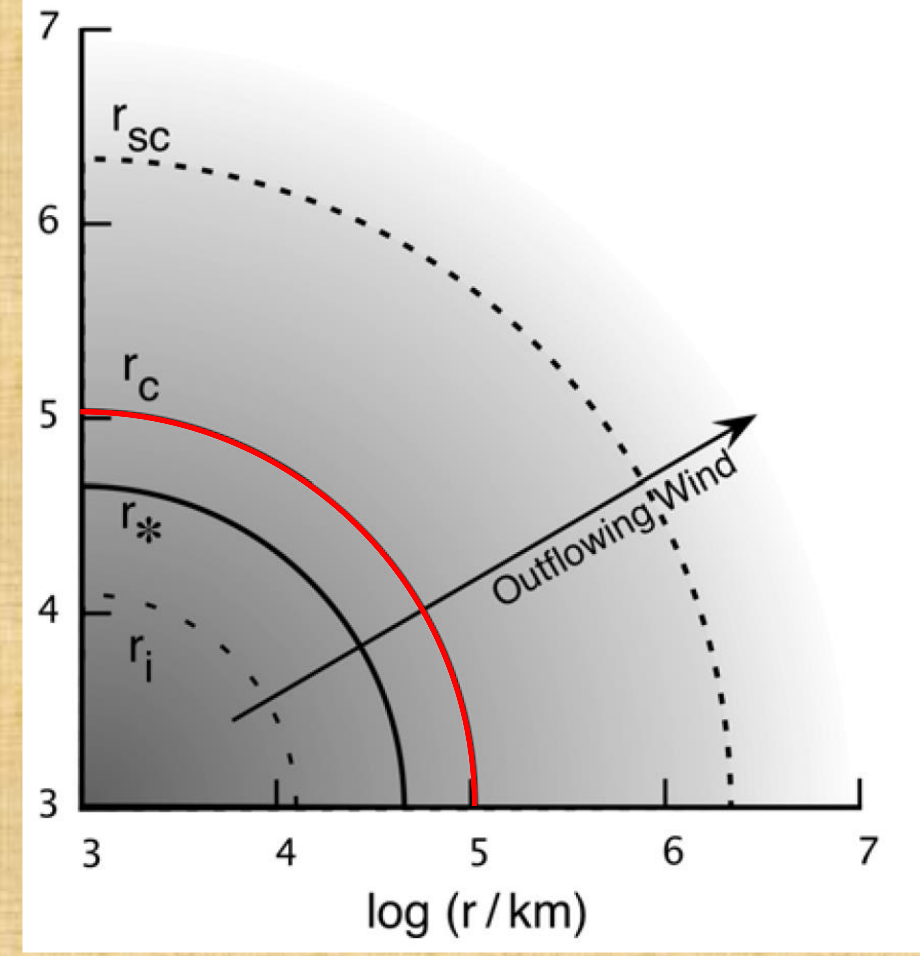
Parameters:
m: black hole mass
$\dot{m}$: normalized accretion rate
α: viscosity parameter

α mainly tells how rapidly the gravitational energy released in the accretion flow at r_i is turned into heat. This ultimately determines the density and wind lift-off velocity at r_i, and it sets the stage for the ensuing wind structure.

Physical Structure – Intro

The Singular/Critical Radius (r_c):

Similar to what we described earlier, when the wind passes this point, it becomes supersonic and thus kinetically dominated.



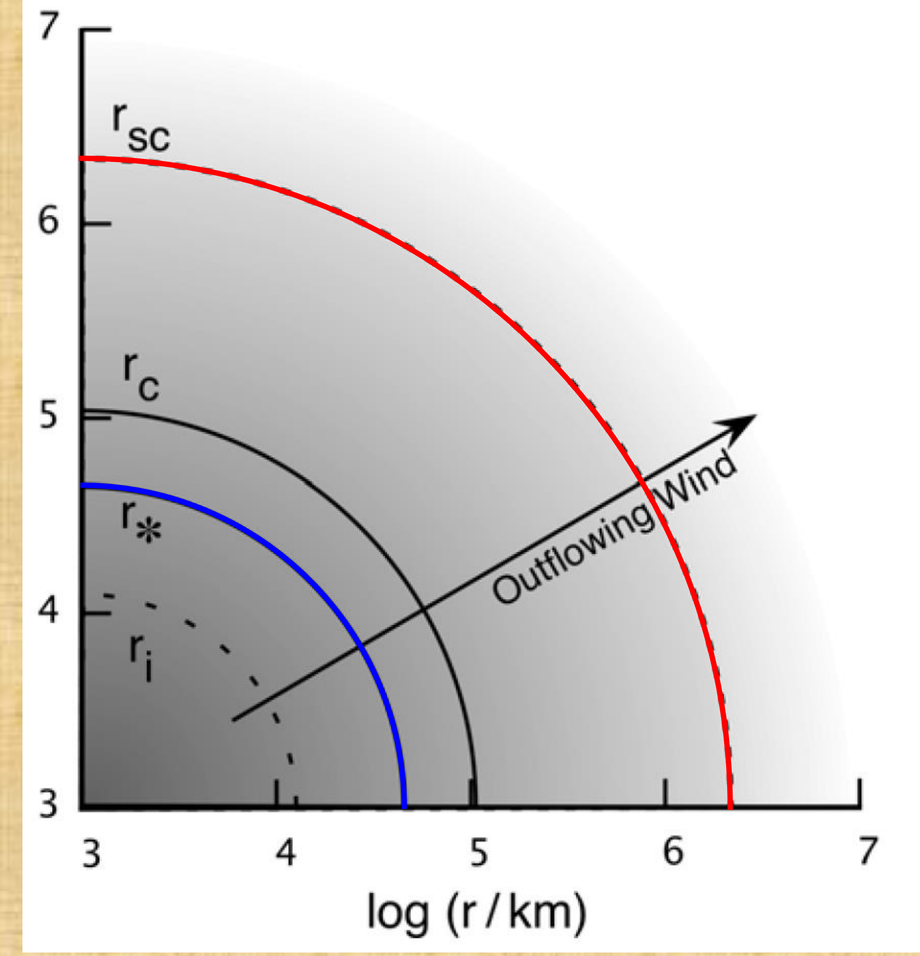
Beyond r_c the flow becomes increasingly difficult to accelerate, because its speed is so fast that it leaves the system before dynamical forces have much time to act upon it.

The wind achieves a terminal velocity not much greater than the critical velocity, so $v_{ter} \sim v_c$.

Physical Structure – Intro

The Scattersphere Radius (r_{sc}):

The wind becomes optically thin to electron scattering at this radius.



Most models that we will describe here will do so in the supersonic portion of the flow; that is $r_{\mathrm{sc}} > r_{\mathrm{c}}$.

r_* is the photosphere radius defined to be the surface of last absorption. (The word photosphere to my knowledge is usually used for surface of last scattering or absorption?)

However, as we discussed in previous weeks, electron scattering dominates in accretion disks and therefore $r_* < r_{sc}$.

Basic Equations

Even though there may be some initial departure from spherical symmetry near the injection region, generally the wind will be spherically symmetric throughout most of its structure. Considering non-relativistic winds, we have the following equations:

$$\frac{1}{r^2}\frac{d(r^2\rho v)}{\mathrm{dr}} = 0$$ Mass continuity

$$v\frac{\mathrm{dv}}{\mathrm{dr}} = -\frac{GM}{r^2} - \frac{1}{\rho}\frac{\mathrm{dp}_g}{\mathrm{dr}} + \frac{\kappa_{\mathrm{es}} H_r}{c}$$ Momentum equation

Recall $a_r^{\mathrm{lines}} = \frac{\kappa_{\mathrm{lines}} L_{\mathrm{rad}}}{4\pi c r^2}$

$$v\frac{\mathrm{d}\varepsilon}{\mathrm{dr}} + \frac{(p+\varepsilon)}{r^2}\frac{d(r^2 v)}{\mathrm{dr}} = -\frac{1}{r^2}\frac{d(r^2 H_r)}{\mathrm{dr}} + \rho\dot{q}$$ Energy equation

v: The outward wind velocity

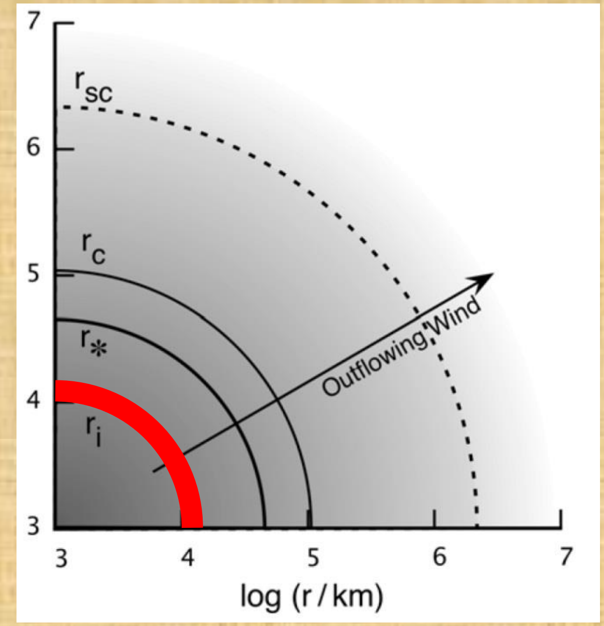
$\varepsilon = \varepsilon_r + \varepsilon_g \approx \varepsilon_r$: the energy density of the wind (dominated by photons)

$p = p_r + p_g \approx p_r$: pressure of the wind (also dominated by photons)

$H_r = L_{\mathrm{rad}}/4\,\pi r^2$: outward flux of radiation

$\varepsilon \approx 3p$: Relation for photons

The injection region



$$\rho\dot{q} = \begin{cases} \alpha\frac{(p+\varepsilon)}{\tau_{\text{dyn}}} & r \le r_i \\ 0 & r > r_i \end{cases} \qquad L_T \equiv \int 4\pi r^2 \rho\dot{q}\,dr \approx 4\pi r_i^{\,3}\alpha\frac{(p+\varepsilon)}{\tau_{\text{dyn}}}$$

$$\frac{1}{r^2}\frac{d(r^2\rho v)}{dr} = 0 \rightarrow \Delta\dot{M} \equiv 4\pi r_i^{\,2}\rho_i v_i$$

$$v\frac{dv}{dr} = -\frac{GM}{r^2} - \frac{1}{\rho}\frac{dp_g}{dr} + \frac{\kappa_{es}H_r}{c} \rightarrow L_{\text{rad}} = \left(1 - \frac{p_g}{p}\right)L_{\text{Edd}}$$

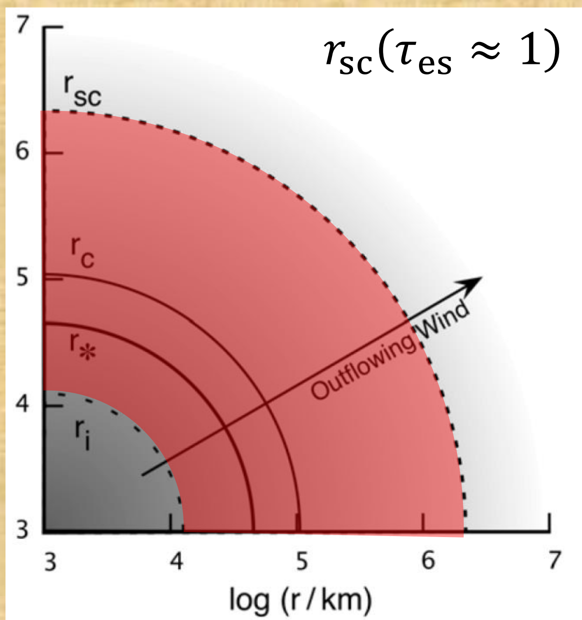
$$v\frac{d\varepsilon}{dr} + \frac{(p+\varepsilon)}{r^2}\frac{d(r^2 v)}{dr} = -\frac{1}{r^2}\frac{d(r^2 H_r)}{dr} + \rho\dot{q} \rightarrow \Delta\dot{M}\left(\frac{p+\varepsilon}{\rho}\right) = L_T - L_{\text{rad}}$$

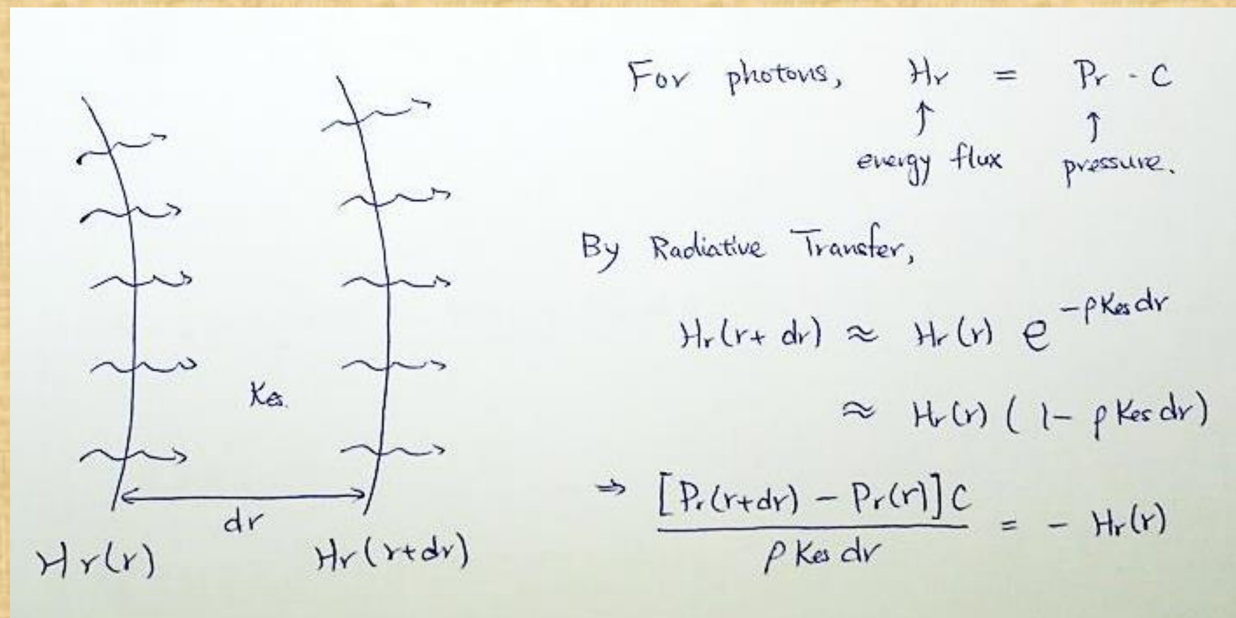
If L_{rad} is close to L_{Edd}, $\Delta\dot{M} = (L_T - L_{\text{rad}})\left(\frac{\rho}{p+\varepsilon}\right)$

the wind mass-loss rate is just enough to carry off the remaining enthalpy generated above the Eddington luminosity

Diffusion approximation $H_r = \frac{-c}{\rho\kappa_{es}} \frac{dp_r}{dr}$

Within the region where it is optically thick for photons ($r < r_{sc}$), we can use the following diffusion approximation:





This then modifies the momentum equation to $v\frac{dv}{dr} = -\frac{GM}{r^2} - \frac{1}{\rho}\frac{dp_g}{dr} + \frac{\kappa_{es}H_r}{c} = -\frac{GM}{r^2} - \frac{1}{\rho}\frac{dp}{dr}$

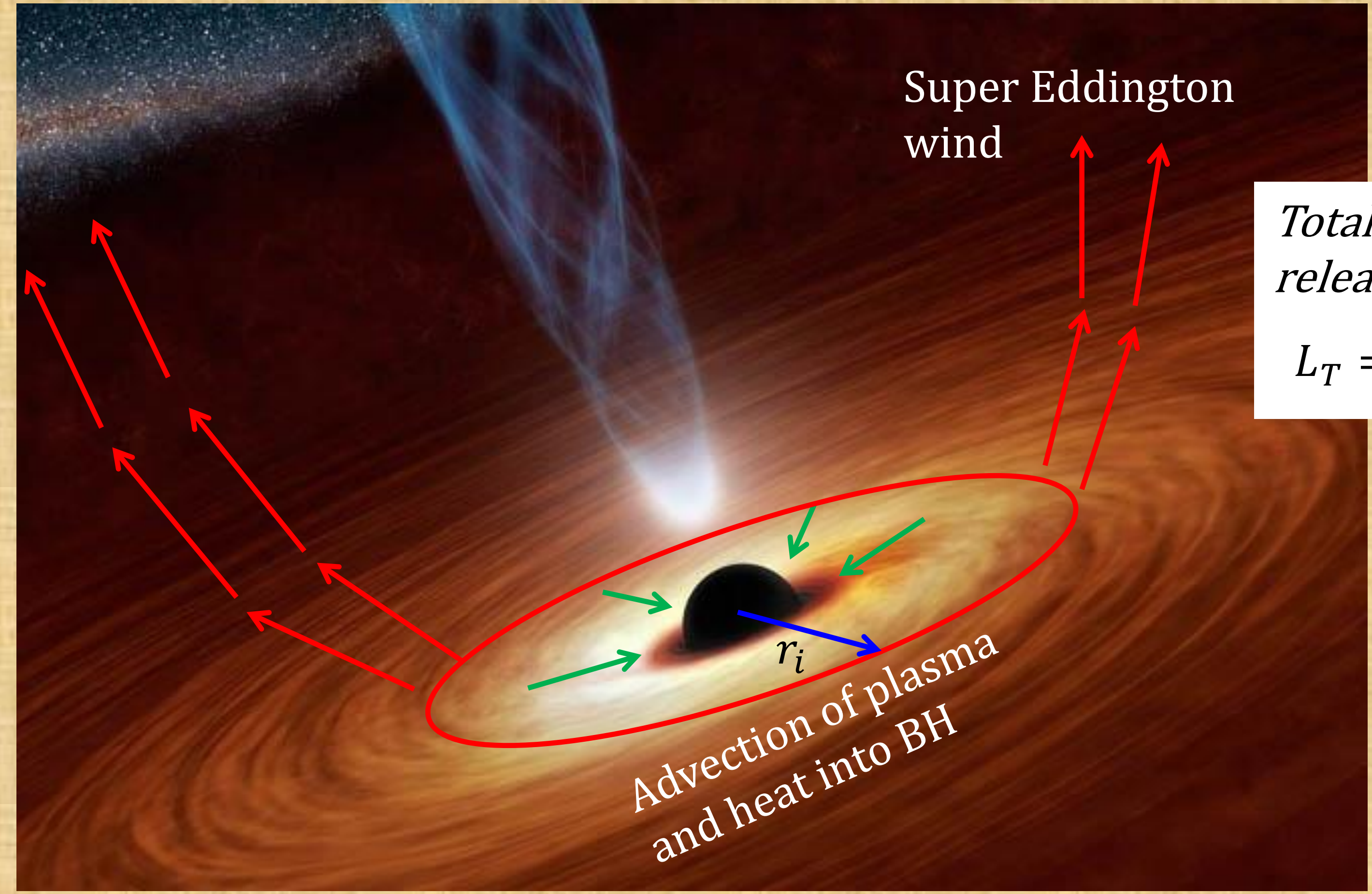
This gives, approximately $\frac{p}{\rho} \approx \frac{GM}{4r}$ HOW!?

$$\Delta\dot{M} = (L_T - L_{rad})\left(\frac{\rho}{p+\varepsilon}\right) = \frac{4\pi c r_i}{\kappa_{es}}\left(\frac{L_T}{L_{Edd}} - 1\right) \equiv \dot{M}\left(\frac{L_T}{L_{Edd}} - 1\right)$$

The total accreted mass $\dot{M}$

For simplicity, we consider only two parts: (below is only for illustration, remember that it's optically thick so most likely you aren't seeing any of this and it's spherical…)

1. Matter that falls in to r_i then is blown off $\Delta\dot{M}$ (releases $(GM/r_i)\Delta\dot{M}$ of potential energy)
2. Matter that falls into the hole $\delta\dot{M}$ (energy released $\epsilon_{\rm acc}\delta\dot{M}c^2$)



Simple relation:

$$\dot{M} = \delta\dot{M} + \Delta\dot{M}$$

Total Luminosity that is released is thus:

$$L_T = \epsilon_{\rm acc}\delta\dot{M}c^2 + \frac{\rm GM}{r^i}\Delta\dot{M}$$

Two possible solutions

$$\Delta\dot{M} \equiv \dot{M}\left(\frac{L_T}{L_{\rm Edd}} - 1\right) \qquad \dot{M} = \delta\dot{M} + \Delta\dot{M} \qquad L_T = \epsilon_{\rm acc}\delta\dot{M}c^2 + \frac{\rm GM}{r^i}\Delta\dot{M}$$

	No-wind solution	Super-Eddington wind solution
L_T	$\epsilon_{\rm acc}\delta\dot{M}c^2$	$\left(1 + \frac{r_g}{\epsilon_{\rm acc} r_i}(\dot{m} - 1)\right)L_{\rm Edd} = \left(2 - \frac{1}{\dot{m}}\right)L_{\rm Edd}$
$\Delta\dot{M}$	0	$\dot{M} - \dot{M}_{\rm Edd}$
$\delta\dot{M}$	$\dot{M}$	$\dot{M}_{\rm Edd}$

When a super-Eddington wind is produced, only one $\dot{M}_{\rm Edd}$ can finally accrete onto the black hole. The rest of the original accretion rate emerges in a strong wind, and does so from the rather large radius $r_i \approx \dot{m}\, r_{\rm g}/\epsilon_{\rm acc}$. For $\dot{m} \gg 1$ only *two* Eddington luminosities of power are generated, one to produce the photon radiation and one that is advected outward as enthalpy in the wind. Equations (13.13) to (13.15) now allow us to solve for the conditions in the injection region. The solutions will be given below with the rest of the subsonic structure.

The Wind Structure

Solving the following equations, for the conditions explained below, we can get the following limiting cases (I will not explain the derivation as it took a whole afternoon…)

$$\frac{1}{r^2}\frac{d(r^2\rho v)}{\mathrm{dr}} = 0 \qquad v\frac{\mathrm{dv}}{\mathrm{dr}} = -\frac{GM}{r^2} - \frac{1}{\rho}\frac{\mathrm{dp}}{\mathrm{dr}} \qquad v\frac{\mathrm{d}\varepsilon}{\mathrm{dr}} + \frac{(p+\varepsilon)}{r^2}\frac{d(r^2 v)}{\mathrm{dr}} = -\frac{1}{r^2}\frac{d(r^2 H_r)}{\mathrm{dr}} + \rho\dot{q}$$

Region	$r < r_c$	$r > r_c$
Approximation	$v \ll c_s$	$v\frac{\mathrm{dv}}{\mathrm{dr}} \approx 0$
Argument	When the wind has just been launched, it is still way below the (total) sound speed.	(Far?) Beyond the critical point, the inertial terms dominate and the wind is no longer accerated.
ρ	$\rho_i\left(\frac{r}{r_i}\right)^{-3}$	$\rho_c\left(\frac{r}{r_c}\right)^{-2} = \rho_i\zeta^{-3}\left(\frac{r}{r_c}\right)^{-2}$
v	$v_i\left(\frac{r}{r_i}\right)$	$\zeta \equiv \frac{r_c}{r_i}$ $\quad v_c = v_i\,\zeta$
p	$p_i\left(\frac{r}{r_i}\right)^{-4}$	$p_c\left(\frac{r}{r_c}\right)^{-3} = p_i\zeta^{-4}\left(\frac{r}{r_c}\right)^{-3}$

← I can't get this..

Along with the previously derived parameters, the following numerical values could be found (at least so says the book…, I didn't have time to work them out)

	No-wind solution	Super-Eddington wind solution
L_T	$\epsilon_{\mathrm{acc}}\delta\dot{M}c^2$	$\left(1+\frac{r_g}{\epsilon_{\mathrm{acc}}r_i}(\dot{m}-1)\right)L_{\mathrm{Edd}} = \left(2-\frac{1}{\dot{m}}\right)L_{\mathrm{Edd}}$
$\Delta\dot{M}$	0	$\dot{M}-\dot{M}_{\mathrm{Edd}}$
$\delta\dot{M}$	$\dot{M}$	$\dot{M}_{\mathrm{Edd}}$

	ρ	v	p
$r < r_c$	$\rho_i\left(\frac{r}{r_i}\right)^{-3}$	$v_i\left(\frac{r}{r_i}\right)$	$p_i\left(\frac{r}{r_i}\right)^{-4}$
$r > r_c$	$\rho_c\left(\frac{r}{r_c}\right)^{-2} = \rho_i\zeta^{-3}\left(\frac{r}{r_c}\right)^{-2}$	$v_c = v_i\zeta$	$p_c\left(\frac{r}{r_c}\right)^{-3} = p_i\zeta^{-4}\left(\frac{r}{r_c}\right)^{-3}$

$r < r_c$

r_i	$=$	8.3×10^{5}	cm		m	$\dot{m}$	
ρ	$=$	2.1×10^{-5}	$\mathrm{g\,cm^{-3}}$	α^{-1}	m^{-1}	$\dot{m}^{-1/2}$	z^{-3}
p	$=$	2.3×10^{15}	dyn	α^{-1}	m^{-1}	$\dot{m}^{-3/2}$	z^{-4}
V_Z	$=+$	9.2×10^{9}	$\mathrm{cm\,s^{-1}}$	α		$\dot{m}^{-1/2}$	z
τ_{es}	$=$	5.5		α^{-1}		$\dot{m}^{1/2}$	z^{-2}
$\Delta\dot{M}$	$=$	1.4×10^{18}	$\mathrm{g\,s^{-1}}$		m	$\dot{m}$	
$L_{\mathrm{acc,wind}}$	$=$	2.1×10^{38}	$\mathrm{erg\,s^{-1}}$		m		

$r > r_c$

r_c	$=$	1.17×10^{6}	cm	$\alpha^{-1/2}$	m	$\dot{m}$	
ρ	$=$	1.48×10^{-5}	$\mathrm{g\,cm^{-3}}$	$\alpha^{-1/2}$	m^{-1}	$\dot{m}^{-1/2}$	z^{-2}
p	$=$	1.63×10^{15}	dyn	$\alpha^{-1/2}$	m^{-1}	$\dot{m}^{-3/2}$	z^{-3}
V_Z	$=+$	1.30×10^{10}	$\mathrm{cm\,s^{-1}}$	$\alpha^{1/2}$		$\dot{m}^{-1/2}$	
τ_{es}	$=$	3.9		$\alpha^{-1/2}$		$\dot{m}^{1/2}$	z^{-1}
r_{sc}	$=$	3.2×10^{6}	cm	$\alpha^{-1/2}$	m	$\dot{m}^{3/2}$	

Now, it's critical to find where the critical point (r_c) is!

Before we can go into this, we need to first look at two (again, limiting) cases of the energy equation as it will affect how the critical point should be determined!

The energy equation $v\frac{\mathrm{d}\varepsilon}{\mathrm{dr}} + \frac{(p+\varepsilon)}{r^2}\frac{d(r^2 v)}{\mathrm{dr}} = -\frac{1}{r^2}\frac{d(r^2 H_r)}{\mathrm{dr}}$

There can be two extremes as listed below:

	Radiative	Adiabatic
Basic Equation	$\frac{1}{r^2}\frac{d(r^2 H_r)}{\mathrm{dr}} = 0$	$v\frac{\mathrm{d}\varepsilon}{\mathrm{dr}} + \frac{(p+\varepsilon)}{r^2}\frac{d(r^2 v)}{\mathrm{dr}} = 0$
Explanation	Photon diffusion dominates over the force of electrons trying to drag the photons along adiabatically	The photons are dragged along with the electrons therefore there is no net flux between different radii
$\frac{d\ell\mathrm{n}(V)}{d\ell\mathrm{n}(r)}$	$-\left(\frac{(1-\ell_{\mathrm{rad}})\left(\frac{\mathrm{GM}}{r}\right)-2a^2+\frac{da^2}{d\ell\mathrm{n}(r)}}{v^2-a^2}\right)$	$-\left(\frac{\left(\frac{\mathrm{GM}}{r}\right)-2{c_s}^2}{v^2-{c_s}^2}\right)$
Critical point r_c	$r_{s,g}$ (gas pressure sonic point)	r_s (total pressure sonic point)
Velocity at r_c	$a = \sqrt{\frac{p_g}{\rho}}$	$c_s = \sqrt{\frac{\gamma \mathrm{p}}{\rho}}$

Radiative or Adiabatic?

Now we are faced with a different problem: The radiative and adiabatic cases have different critical radii.

Therefore it now becomes important to know under what conditions will each be a valid approximation.

The book gives only (again) limiting cases and slices them apart with a very stiff boundary.

The energy equation $v\,\underline{\frac{\mathrm{d}\varepsilon}{\mathrm{d}r}} + \frac{(p+\varepsilon)}{r^2}\frac{d(r^2 v)}{\mathrm{d}r} = \underline{-\frac{1}{r^2}\frac{d(r^2 H_r)}{\mathrm{d}r}}$

The question: which flux dominates?

Radiative or Adiabatic? (cont')

The energy equation $v\frac{\mathrm{d}\varepsilon}{\mathrm{dr}} + \frac{(p+\varepsilon)}{r^2}\frac{d(r^2 v)}{\mathrm{dr}} = -\frac{1}{r^2}\frac{d(r^2 H_r)}{\mathrm{dr}}$

The question: which flux dominates?

The 1st term (Adiabatic dominance):

$$|v\frac{\mathrm{d}\epsilon}{\mathrm{dr}}| \approx \frac{12vp}{r}$$

The 2nd term (Radiative dominance):

$$|\frac{1}{r^2}\frac{d(r^2 H_r)}{\mathrm{dr}}| = |\frac{c}{\kappa_{\mathrm{es}} r^2}[\frac{d}{\mathrm{dr}}\left(r^2 v\frac{\mathrm{dv}}{\mathrm{dr}}\right) + \frac{d}{\mathrm{dr}}\left(\frac{r^2}{\rho}\frac{\mathrm{dp}_g}{\mathrm{dr}}\right)]| \approx \begin{cases} \frac{3c}{\kappa_{\mathrm{es}} r^2} a^2 & v^2 < a^2 \\ \frac{3c}{\kappa_{\mathrm{es}} r^2} v^2 & v^2 > a^2 \end{cases}$$

$v\frac{\mathrm{dv}}{\mathrm{dr}} = -\frac{GM}{r^2} - \frac{1}{\rho}\frac{\mathrm{dp}_g}{\mathrm{dr}} + \frac{\kappa_{\mathrm{es}} H_r}{c}$ The momentum equation

*Note that we can't plug in the diffusion approximation because we don't know yet whether this transition is at the optically thin or thick region.

Radiative or Adiabatic? (cont')

*I see no point in retyping this this I'll have to read it through anyway…

Therefore, the radius $r_{\rm ad}$ of transition from adiabatic to radiative flow depends on its location relative to the two sonic points (*i.e.*, how fast the wind is expanding locally):

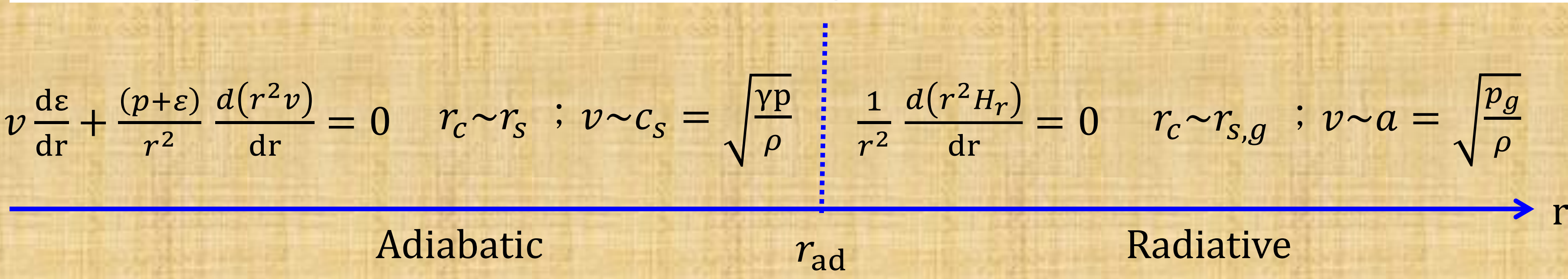
$r_{\rm tr}$ is the trapping radius introduced earlier in the book but to save time, I won't introduce it here.

$$
\begin{aligned}
r_{\rm ad} &= \frac{c_s^2}{a^2}\, r_{\rm tr} & & r_{\rm ad} < r_{s,\rm g} \\
&= \frac{c_s^2}{V^2}\, r_{\rm tr} & & r_{s,\rm g} < r_{\rm ad} < r_s \\
&= r_{\rm tr} & & r_s < r_{\rm ad}
\end{aligned}
\qquad (13.24)
$$

which is good to within factors of order unity.

$$v\frac{d\varepsilon}{dr} + \frac{(p+\varepsilon)}{r^2}\frac{d(r^2 v)}{dr} = 0 \qquad r_c \sim r_s \;;\; v \sim c_s = \sqrt{\frac{\gamma p}{\rho}}$$

$$\frac{1}{r^2}\frac{d(r^2 H_r)}{dr} = 0 \qquad r_c \sim r_{s,g} \;;\; v \sim a = \sqrt{\frac{p_g}{\rho}}$$



Different cases

Optically thin ($r_c > r_{\rm sc}$)

Case A ($r_{\rm ad} < r_{\rm sc}$ from book, $r_{\rm ad} < r_{\rm sc}$ from paper… I think the book is mistyped)

Optically thick ($r_c < r_{\rm sc}$)

Case B $\quad r_{\rm ad} < r_{\rm s,g} < r_{\rm sc} < r_{\rm s}$

Case C1 $\quad r_{s,g} < r_{\rm ad} < r_s; r_{\rm ad} < r_{\rm sc}$

Case C2 $\quad r_{\rm sg} < r_{\rm sc} < r_{\rm ad} < r_s$

Case D $\quad r_s < r_{\rm ad} < r_{\rm sc}$

	ρ	v	p
$r < r_c$	$\rho_i \left(\frac{r}{r_i}\right)^{-3}$	$v_i \left(\frac{r}{r_i}\right)$	$p_i \left(\frac{r}{r_i}\right)^{-4}$
$r > r_c$	$\rho_c \left(\frac{r}{r_c}\right)^{-2} = \rho_i \zeta^{-3} \left(\frac{r}{r_c}\right)^{-2}$	$v_c = v_i \zeta$	$p_c \left(\frac{r}{r_c}\right)^{-3} = p_i \zeta^{-4} \left(\frac{r}{r_c}\right)^{-3}$

$v \frac{\mathrm{d}\varepsilon}{\mathrm{dr}} + \frac{(p+\varepsilon)}{r^2} \frac{d(r^2 v)}{\mathrm{dr}} = 0 \quad r_c \sim r_s \;;\; v \sim c_s = \sqrt{\frac{\gamma \mathrm{p}}{\rho}}$

$\frac{1}{r^2} \frac{d(r^2 H_r)}{\mathrm{dr}} = 0 \quad r_c \sim r_{s,g} \;;\; v \sim a = \sqrt{\frac{p_g}{\rho}}$

r

Adiabatic $\quad r_{\rm ad} \quad$ Radiative

Optically thin Case A ($r_{sc} < r_{sg}$)

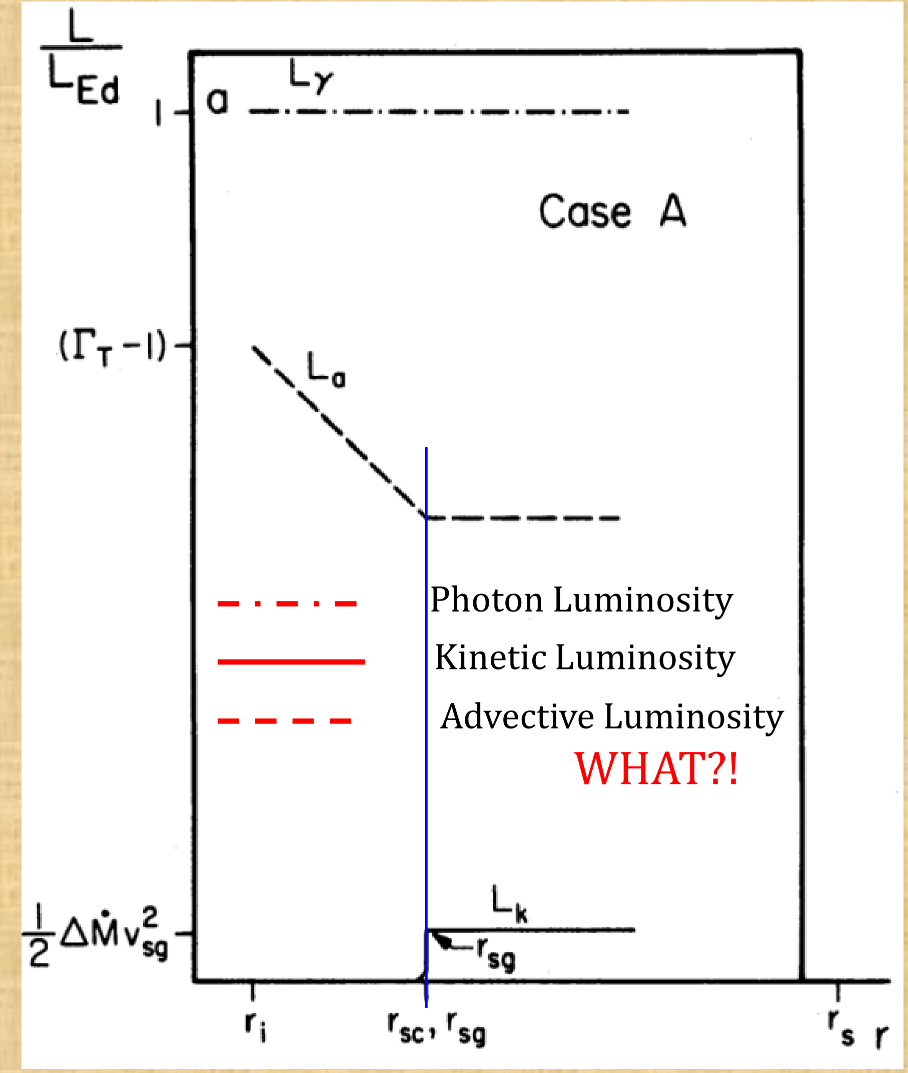
The wind reaches an optically thin region r > r_{sc} before it even gets to $r_{s,g}$.

This simply reduces to the case like our early discussion of the line driven wind which gives $v_{ter} \sim a(r_{s,g})$

	ρ	v	p
$r < r_c$	$\rho_i \left(\frac{r}{r_i}\right)^{-3}$	$v_i \left(\frac{r}{r_i}\right)$	$p_i \left(\frac{r}{r_i}\right)^{-4}$
$r > r_c$	$\rho_c \left(\frac{r}{r_c}\right)^{-2}$	$v_c = v_i \zeta$	$p_c \left(\frac{r}{r_c}\right)^{-3}$

$r_c \sim r_s$; $v \sim c_s = \sqrt{\frac{\gamma p}{\rho}}$ $\quad$ $r_c \sim r_{s,g}$; $v \sim a = \sqrt{\frac{p_g}{\rho}}$

Adiabatic $\quad$ r_{ad} $\quad$ Radiative $\quad$ r



Optically Thick Case B ($r_{\rm ad} < r_{\rm s,g} < r_{\rm sc} < r_{\rm s}$)

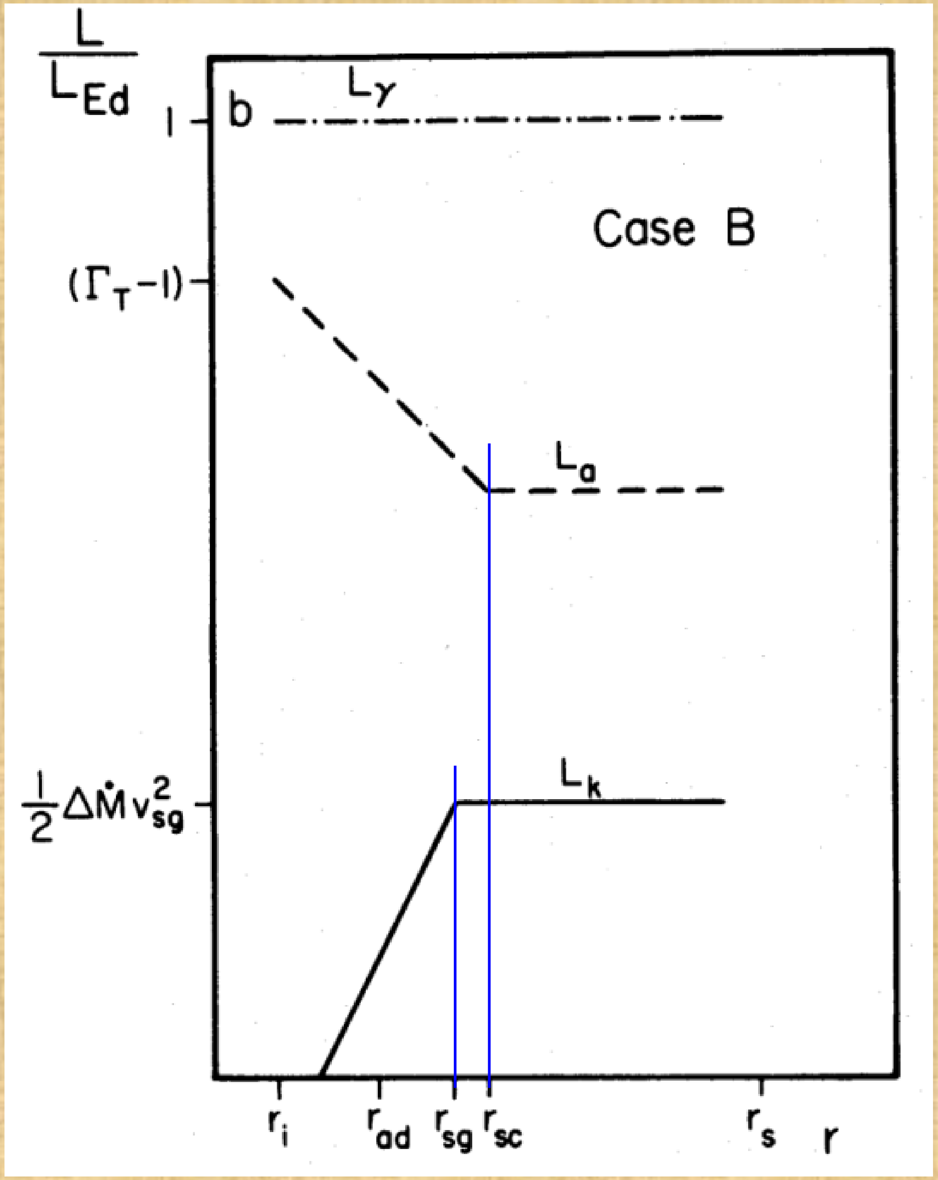
$r_{\rm ad}$ being smallest tells us that we are in the radiative zone. Therefore the flow will accelerate to around the gas pressure sonic point $r_{\rm s,g}$. Giving $v_{\rm ter} \sim a(r_{s,g})$

	ρ	v	p
$r < r_c$	$\rho_i \left(\frac{r}{r_i}\right)^{-3}$	$v_i \left(\frac{r}{r_i}\right)$	$p_i \left(\frac{r}{r_i}\right)^{-4}$
$r > r_c$	$\rho_c \left(\frac{r}{r_c}\right)^{-2}$	$v_c = v_i \zeta$	$p_c \left(\frac{r}{r_c}\right)^{-3}$

$r_c \sim r_s$; $v \sim c_s = \sqrt{\frac{\gamma p}{\rho}}$ | $r_c \sim r_{s,g}$; $v \sim a = \sqrt{\frac{p_g}{\rho}}$

→ r

Adiabatic | $r_{\rm ad}$ | Radiative



Optically Thick Case D ($r_s < r_{\rm ad} < r_{\rm sc}$)

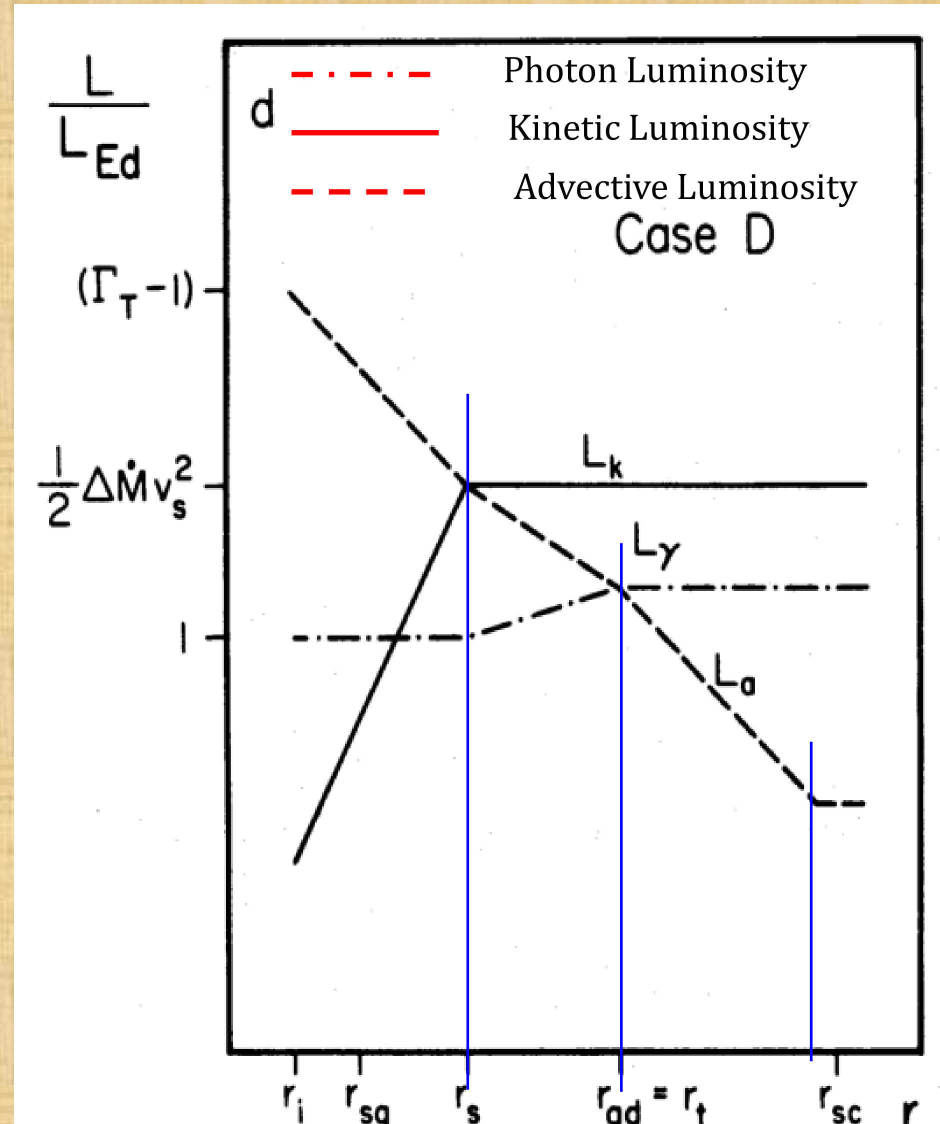
The wind is fully accelerated within the adiabatic regime, reaching critical point at r_s with terminal velocity $v_{\rm ter} \sim c_s(r_s)$

These winds also have enormous mass-loss rates, very high $L_T/L_{\rm Edd}$, and even sper-Eddington photon luminosities ($L_{\rm rad} > L_{\rm Edd}$)

	ρ	v	p
$r < r_c$	$\rho_i \left(\frac{r}{r_i}\right)^{-3}$	$v_i \left(\frac{r}{r_i}\right)$	$p_i \left(\frac{r}{r_i}\right)^{-4}$
$r > r_c$	$\rho_c \left(\frac{r}{r_c}\right)^{-2}$	$v_c = v_i \zeta$	$p_c \left(\frac{r}{r_c}\right)^{-3}$

$r_c \sim r_s$; $v \sim c_s = \sqrt{\frac{\gamma p}{\rho}}$ $\quad$ $r_c \sim r_{s,g}$; $v \sim a = \sqrt{\frac{p_g}{\rho}}$

Adiabatic $\quad r_{\rm ad} \quad$ Radiative $\quad$ r



Optically thick Case C1 ($r_{s,g} < r_{\text{ad}} < r_s; r_{\text{ad}} < r_{\text{sc}}$)

For super-Eddington accretion flows onto black holes that produce winds, the strange and interesting Case C is, by far, the most common wind structure. In this case, the critical point is at r_{ad}.

Argument:

1. r_{c} can't be at $r_{s,g}$ because $r_{s,g} < r_{\text{ad}}$ means that the wind would accelerate pass $r_{s,g}$ within the Adiabatic region (which has $r_c \sim r_s$) .
2. r_{c} can't be at r_s as $r_{\text{ad}} < r_s$ says that the wind is radiative when it gets to r_s.

	ρ	v	p
$r < r_c$	$\rho_i \left(\frac{r}{r_i}\right)^{-3}$	$v_i \left(\frac{r}{r_i}\right)$	$p_i \left(\frac{r}{r_i}\right)^{-4}$
$r > r_c$	$\rho_c \left(\frac{r}{r_c}\right)^{-2}$	$v_c = v_i \zeta$	$p_c \left(\frac{r}{r_c}\right)^{-3}$

Adiabatic, before critical point

$v \propto r$

Radiative, passed critical point

$v \sim \text{const.}$

$r_{s,g}$ r_{ad} r_s

$r_c \sim r_s \;;\; v \sim c_s = \sqrt{\frac{\gamma p}{\rho}}$

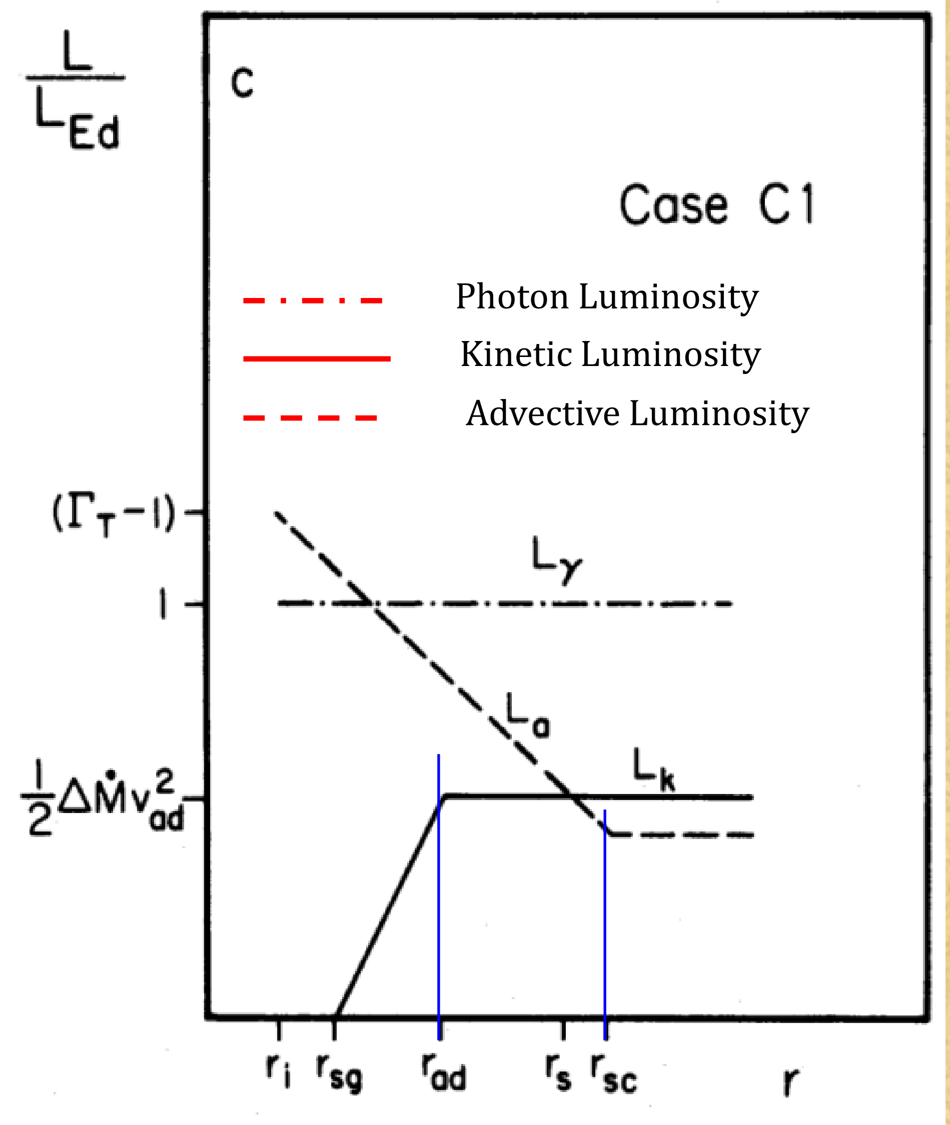
$r_c \sim r_{s,g} \;;\; v \sim a = \sqrt{\frac{p_g}{\rho}}$

r

Adiabatic r_{ad} Radiative

The critical radius should be somewhere around r_{ad}!

Optically thick Case C1 ($r_{s,g} < r_{\rm ad} < r_s; r_{\rm ad} < r_{\rm sc}$)



	ρ	v	p
$r < r_c$	$\rho_i\left(\frac{r}{r_i}\right)^{-3}$	$v_i\left(\frac{r}{r_i}\right)$	$p_i\left(\frac{r}{r_i}\right)^{-4}$
$r > r_c$	$\rho_c\left(\frac{r}{r_c}\right)^{-2}$	$v_c = v_i\zeta$	$p_c\left(\frac{r}{r_c}\right)^{-3}$

$r_c \sim r_s$; $v \sim c_s = \sqrt{\frac{\gamma p}{\rho}}$

$r_c \sim r_{s,g}$; $v \sim a = \sqrt{\frac{p_g}{\rho}}$

r

Adiabatic $r_{\rm ad}$ Radiative

Optically Thick Case C2 ($r_{sg} < r_{sc} < r_{ad} < r_s$)

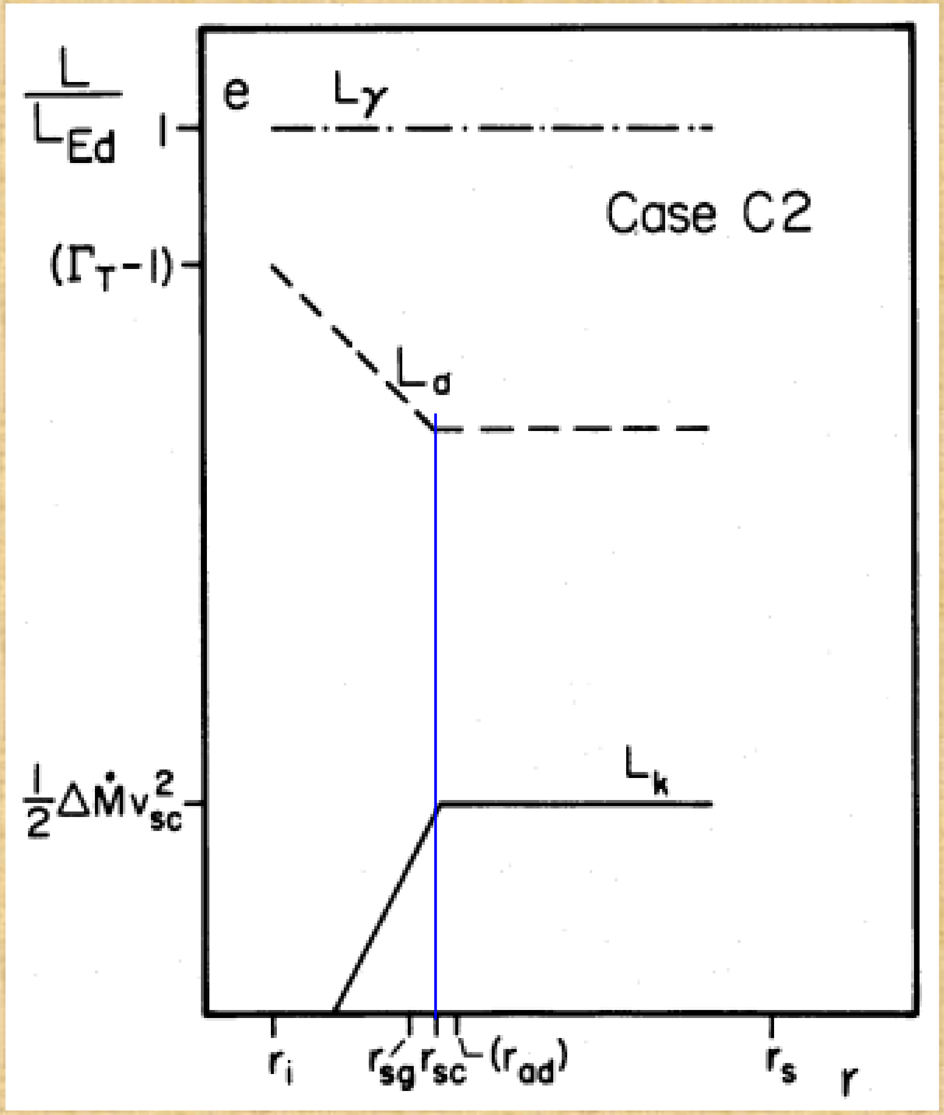
	ρ	v	p
$r < r_c$	$\rho_i \left(\frac{r}{r_i}\right)^{-3}$	$v_i \left(\frac{r}{r_i}\right)$	$p_i \left(\frac{r}{r_i}\right)^{-4}$
$r > r_c$	$\rho_c \left(\frac{r}{r_c}\right)^{-2}$	$v_c = v_i \zeta$	$p_c \left(\frac{r}{r_c}\right)^{-3}$

$r_c \sim r_s$; $v \sim c_s = \sqrt{\frac{\gamma p}{\rho}}$

$r_c \sim r_{s,g}$; $v \sim a = \sqrt{\frac{p_g}{\rho}}$

r

Adiabatic r_{ad} Radiative



ApJ, 289, 634 (1985)

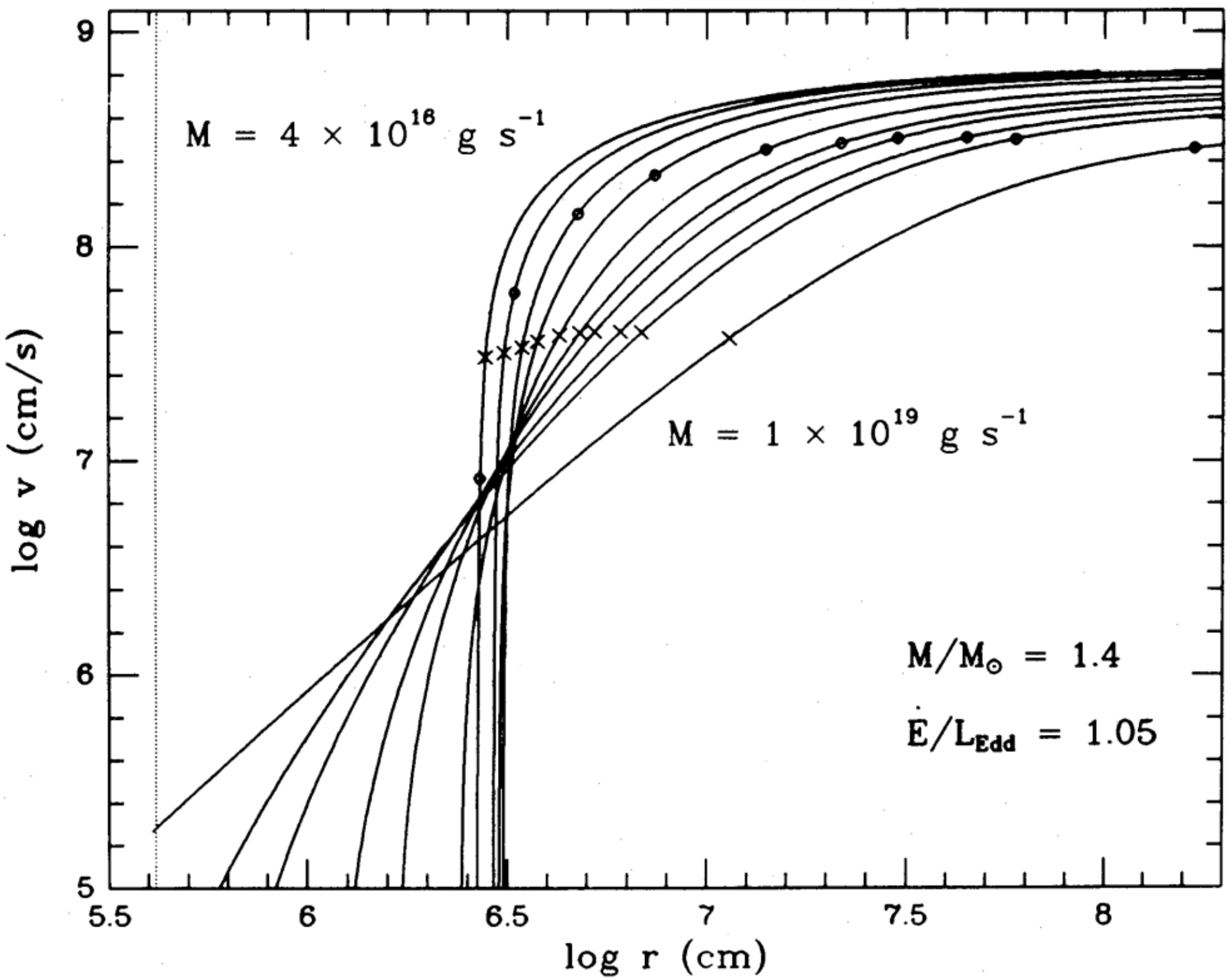


FIG. 5.—Variation of outflow velocity with radius for the same models as in Fig. 3. The symbols are the same. The dotted vertical line marks the gravitational radius.

	No-wind solution	Super-Eddington wind solution
L_T	$\epsilon_{\rm acc}\delta\dot{M}c^2$	$\left(1+\frac{r_g}{\epsilon_{\rm acc}r_i}(\dot{m}-1)\right)L_{\rm Edd} = \left(2-\frac{1}{\dot{m}}\right)L_{\rm Edd}$
$\Delta\dot{M}$	0	$\dot{M}-\dot{M}_{\rm Edd}$
$\delta\dot{M}$	$\dot{M}$	$\dot{M}_{\rm Edd}$

	ρ	v	p
$r < r_c$	$\rho_i\left(\frac{r}{r_i}\right)^{-3}$	$v_i\left(\frac{r}{r_i}\right)$	$p_i\left(\frac{r}{r_i}\right)^{-4}$
$r > r_c$	$\rho_c\left(\frac{r}{r_c}\right)^{-2} = \rho_i\zeta^{-3}\left(\frac{r}{r_c}\right)^{-2}$	$v_c = v_i\zeta$	$p_c\left(\frac{r}{r_c}\right)^{-3} = p_i\zeta^{-4}\left(\frac{r}{r_c}\right)^{-3}$

This was derived for Case C !

$r < r_c$

r_i	$=$	8.3×10^{5}	cm		m	$\dot{m}$	
ρ	$=$	2.1×10^{-5}	$\mathrm{g\,cm^{-3}}$	α^{-1}	m^{-1}	$\dot{m}^{-1/2}$	z^{-3}
p	$=$	2.3×10^{15}	dyn	α^{-1}	m^{-1}	$\dot{m}^{-3/2}$	z^{-4}
V_Z	$=+$	9.2×10^{9}	$\mathrm{cm\,s^{-1}}$	α		$\dot{m}^{-1/2}$	z
$\tau_{\rm es}$	$=$	5.5		α^{-1}		$\dot{m}^{1/2}$	z^{-2}
$\Delta\dot{M}$	$=$	1.4×10^{18}	$\mathrm{g\,s^{-1}}$		m	$\dot{m}$	
$L_{\rm acc,wind}$	$=$	2.1×10^{38}	$\mathrm{erg\,s^{-1}}$		m		

$r > r_c$

r_c	$=$	1.17×10^{6}	cm	$\alpha^{-1/2}$	m	$\dot{m}$	
ρ	$=$	1.48×10^{-5}	$\mathrm{g\,cm^{-3}}$	$\alpha^{-1/2}$	m^{-1}	$\dot{m}^{-1/2}$	z^{-2}
p	$=$	1.63×10^{15}	dyn	$\alpha^{-1/2}$	m^{-1}	$\dot{m}^{-3/2}$	z^{-3}
V_Z	$=+$	1.30×10^{10}	$\mathrm{cm\,s^{-1}}$	$\alpha^{1/2}$		$\dot{m}^{-1/2}$	
$\tau_{\rm es}$	$=$	3.9		$\alpha^{-1/2}$		$\dot{m}^{1/2}$	z^{-1}
$r_{\rm sc}$	$=$	3.2×10^{6}	cm	$\alpha^{-1/2}$	m	$\dot{m}^{3/2}$	

Next Week

No Black Hole Astrophysics Report!